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Research Paper

Rethinking the margin period of risk

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ABSTRACT

We describe a new framework for modeling collateralized exposure under an International Swaps and Derivatives Association Master Agreement with a Credit Support Annex. The proposed model captures the legal and operational aspects of default in considerably greater detail than the models currently used by most practitioners while remaining fully tractable and computationally feasible. Specifically, it considers the remedies and suspension rights available within these legal agreements, the firm’s policies in availing itself of these rights and the typical time it takes to exercise them in practice. The inclusion of these effects is shown to produce significantly higher credit exposure for representative portfolios than do the current models. The increase is especially pronounced when the dynamic initial margin is also present.

Keywords: collateralized positions; initial margin; collateralized exposure; bilateral trading relationships.
1 INTRODUCTION

In modeling the exposure of collateralized positions, it is well recognized that a credit default cannot be treated as a one-time event. Rather, the entire sequence of events leading up to and following the default must be considered, from the last successful margin call in advance of the eventual default to the time when the amount of loss becomes known (in industry parlance, “crystallized”). These events unfold over a period of time called the margin period of risk (MPoR).

To properly identify exposures that arise during the MPoR, a detailed understanding of contractual obligations is essential. In this paper, we focus on collateralized exposure under bilateral trading relationships governed by the International Swaps and Derivatives Association (ISDA) International Master Agreement (IMA) and its Credit Support Annex (CSA). The IMA is by far the most common legal contract for bilateral over-the-counter (OTC) derivatives trading, although other agreements are sometimes used (such as the national forms of agreement used in some jurisdictions for domestic trading). We expect our analysis to apply to a broad class of contracts, although model assumptions should be reexamined to confirm that key legal provisions remain substantially the same as the IMA.

We note that the modeling of default exposure and closeout risk arising from a nonzero MPoR length has received a fair amount of attention in the past (see, for example, Brigo et al 2011; Gibson 2005; Pykhtin 2009, 2010), although most past analysis has been conducted under strong simplifying assumptions about trade and margin flows during the MPoR. Here, we use a more detailed framework for legal and operational behavior to refine the classical models for collateralized exposure modeling.

The rest of this paper is organized as follows. First, we outline the fundamentals of variation margin posting in Section 2 and present the classical collateralized exposure model in Section 3. In Section 4, we discuss the full timeline of events that are likely to transpire during a credit default, from both an operational perspective and a legal perspective. This sets the stage for Section 5, where we propose a condensed representation of the timeline suitable for analytical and numerical work. The resulting setup leads to a significantly more nuanced and flexible definition of collateralized trading exposure. As fixing the actual values of model parameters (“calibrating” the MPoR model) requires taking a stance on corporate behavior and operational procedures, Section 6 discusses how such parameterizations may be done in practice, for various levels of overall model prudence and counterparty types.

1 One exception is the conference presentation by Böcker and Schröder (2011), which contains elements of a more detailed framework, including recognition of the role played by cashflows close to the default event.
In the second part of the paper, we flesh out the model in more detail, especially as it pertains to numerical implementation and quantitative comparisons with the classical model. As a starting point, Section 7 first formulates our exposure model in mathematical terms, and highlights the key differences from classical models by means of numerical results computed by brute-force Monte Carlo simulations. Computational techniques permitting efficient model implementation are introduced in Section 8, along with several test results. Section 9 briefly discusses applications for portfolios with risk-based initial margin, and Section 10 concludes the paper.

2 THE FUNDAMENTALS OF THE VARIATION MARGIN

2.1 Basic definitions

In bilateral OTC derivatives trading, it is common for parties to require posting of collateral to mitigate excessive credit exposures. In the ISDA legal framework, the collateral mechanism is specified in the CSA as a combination of two types of margin: initial margin and variation margin. Although we briefly discuss the initial margin in Section 9, we shall primarily focus on the variation margin (VM), a form of collateral that is regularly adjusted based on the changing value of the bilateral portfolio. The VM is calculated and settled through time according to a set of CSA rules that we review in Section 4.

For concreteness, throughout the paper, we consider the exposure of a bank B to a counterparty C with whom B engages in bilateral OTC trading under the IMA/CSA legal framework. We will refer to C as the “defaulting party” and to B as “the bank” or the “nondefaulting party”. All present value and exposure amounts throughout the paper will be calculated from the viewpoint of B. Let the net default-free market value (to B) of the securities portfolio at time $t$ be $V(t)$, and let $A_B(t)$ and $A_C(t)$ be the collateral support amounts stipulated by the CSA to be posted by B and C, respectively. In the absence of initial margin, it is virtually always the case that only one of $A_B$ and $A_C$ is positive, ie, only one party will be required to post margin at a given point in time.

Assuming that collateral is netted (rather than posted by both parties in full and held in segregated accounts or by a third party), the total collateral amount in B’s possession as of time $t$ may be calculated as $c(t) = A_C(t) - A_B(t)$. Assuming also that collateral may be treated as pari passu with the derivatives portfolio itself for the purposes of the bankruptcy claim, it is common to denote the positive part of the

---

2 Normally, both the collateral and the portfolio would be treated together as a senior unsecured claim of B against the bankruptcy estate of C.
difference \( V(t) - c(t) \) as the exposure \( E(t) \):

\[
E(t) = (V(t) - c(t))^+, \quad c(t) = A_C(t) - A_B(t), \tag{2.1}
\]

where we use the notation \( x^+ = \max(x, 0) \). There are several time lags and practical complications that render (2.1) an imprecise measure for default exposure, and we shall refine it substantially later on. In particular, we emphasize that collateral computed from market and portfolio observations at time \( t \) is generally not transferred to B until several days after \( t \).

The type of VM encountered in the CSA is typically designed to broadly track the value of the portfolio between the parties, thereby ensuring that \( E(t) \) in (2.1) does not grow excessive. However, to avoid unnecessary operational expenses, it is common to introduce language in the CSA to relax margin transfer requirements if the amounts are sufficiently small. To that end, the typical CSA language for collateral calculations will stipulate the following:

- collateral posting thresholds by each party, \( h_B \) and \( h_C \), representing the maximum permitted amount of exposure before B or C, respectively, is required to post any collateral;
- a minimum transfer amount (MTA), establishing the minimum valid amount of a margin call;
- rounding, which rounds collateral movements to some reasonable unit (say, US$1000).

Formally, the effects of thresholds on stipulated collateral may be written as

\[
A_B(t) = (-V(t) - h_B)^+, \quad A_C(t) = (V(t) - h_C)^+, \tag{2.2}
\]

with the net stipulated credit support amount assigned to B being \( c(t) = A_C(t) - A_B(t) \) as before. The actual availability of this amount is then subject to the (path-dependent) effects on collateral by MTA and rounding, of which the former has a significant effect only for zero or very small thresholds, and the latter is usually negligible. Both of these effects have been omitted in (2.2).

Most CSAs are bilateral in nature, but unilateral CSAs, in which only one of the two parties is required to post collateral, exist. A CSA may also be formally bilateral but highly asymmetric, requiring both parties to post collateral but with vastly different thresholds (eg, \( h_B = \text{US$20 million} \) versus \( h_C = \text{US$2 million} \)). Typically, even for an asymmetric CSA, the MTA and rounding are the same for both parties.
2.2 Margin calls and cashflows

From an exposure perspective, the frequency with which the amount of collateral is adjusted (the remargining frequency) is a critical component of the CSA. Following the financial crisis, most new IMA/CSAs, especially between major financial institutions, use daily remargining in order to reduce the amount by which exposure can change relative to collateral between margin calls. However, many smaller financial institutions or buy-side clients may not be able to cope with the operational burden of frequent margin calls and will often negotiate remargining frequencies that are weekly, monthly or even longer.

The amount of collateral held by the parties is adjusted to their stipulated values, \( A_B \) and \( A_C \), via the mechanism of a margin call. Many models for exposure treat the margin call as an instantaneous event taking place on the remargining date and completed instantaneously. In reality, the margin call is a chain of events that takes several days to complete. With daily remargining, several such chains are running concurrently in an “interlaced” manner: even as one margin call is waiting to be settled, another may already be initiated. The time lag in this settlement process, along with the inherent lag of the remargining schedule, means that the changes in VM are always running behind the changes in portfolio value. This, in turn, implies that idealized exposure expressions such as (2.1) are inaccurate. The detailed events involved in the initiation and eventual settlement of a margin call will be discussed in Section 4.

With both default settlement and margin transfers being noninstantaneous events, it becomes relevant to track what payment flows take place (or not) during the periods close to a default. Two types of payments are needed here. The first type, for which we will use the term “trade flows”, covers the contractual cashflows, physical settlements and other forms of asset transfers related to the trades themselves.\(^3\) We use the term “trade flows” rather than “cashflows” to emphasize that term sheets may involve flows other than cash, such as transfers of noncash assets (eg, commodities) or physical settlements resulting in the creation of new trades from the old ones (eg, the exercise of a physically settled swaption into a swap). A missed trade flow is a serious event under the IMA, and a “failure to pay” can rapidly lead to default and trade termination unless cured promptly. Any missed trade flow is, of course, part of the nondefaulting party’s claim.

The second type of flow arises from the exchange of collateral between the parties (“margin flows”). The legal treatment of margin flows is governed by the IMA/CSA, rather than by trade documentation between the parties. For our purposes, the most important aspect of the IMA/CSA is the relatively mild treatment it affords a party.

\(^3\) These terms are spelled out in trade documentation and term sheets for each trade.
who misses a margin flow. Indeed, partially missing a margin payment (i.e., paying only a part of the full margin call) is a common occurrence, as disputes about margin amounts happen regularly (and can sometimes persist for years).

During a collateral dispute, the CSA protocol calls for continued payments of the undisputed component of the collateral, but there is of course the possibility that there will be no undisputed component at all, if one party’s counterproposals are sufficiently frivolous. Should suspicions about “gaming” arise, the CSA does contain a methodology to stop disputes through market quotations, but the resulting leakage of position information to competitors is often a strong deterrent to its use. Thus, there is the potential for abuse by firms that are experiencing financial difficulties, and a good possibility that such abuse can go on for some time before B takes further steps to end it. This, in turn, may result in a fairly long period of time between the last fully settled margin call and the eventual termination of the portfolio due to a default.

2.3 Revised exposure definition

In light of the discussion above, let us have a first go at improving (2.1). For this, consider a default of C at time \( \tau \), followed by an early termination of the trade portfolio at time \( t > \tau \). At time \( t \), let \( K(t) \) denote the collateral that B can actually rely on for portfolio termination; this amount will very likely differ from the CSA-stipulated amount \( c(t) \) (and from \( c(\tau) \), for that matter) due to margin transfer time lags and some degree of nonperformance by C. In addition, it is possible that some trade flows were missed; let us denote their value at time \( t \), including accrued interest, as UTF\( (t) \). Then we may redefine exposure generated by a default at time \( \tau \) as

\[
E(t) = (V(t) + \text{UTF}(t) - K(t))^+.
\]  

(2.3)

Note that (2.3) anchors exposure at the termination date, rather than at the default date \( \tau \); we return to this topic in Section 5.3. For later use we also define the time-zero expectation of future, time-\( t \) exposure as

\[
\mathbb{E}E(t) = \mathbb{E}_0(E(t)),
\]

where \( \mathbb{E} \) is the expectations operator in a relevant probability measure.

Determining how \( K(t) \) may differ from \( c(t) \), and how large UTF\( (t) \) can realistically be, will require a more detailed understanding of the settlement and margin processes, a topic we return to in Section 4. First, however, we examine how classical approaches go about modeling \( K(t) \) and UTF\( (t) \) in (2.3).
3 CLASSICAL MODEL FOR COLLATERALIZED EXPOSURE

3.1 Assumptions about margin flows

A naive, and now outdated, model for collateralized exposure follows (2.1) literally, and assumes that the available collateral is exactly equal to its prescribed value at time $t$. That is, in the language of (2.3), we assume $K(t) = c(t)$. In addition, the parties are assumed to pay all of the trade flows as prescribed ($UTF(t) = 0$), and it is assumed that the termination date $t$ in (2.3) equals the default time $\tau$, i.e., there is no lag between the default date and the termination date. In this model, the amount of loss crystallized at time $t$ is a function of portfolio value at a single time point, $V(t)$, and does not depend on the earlier history of $V(\cdot)$. In the limit of “perfect CSA”, where $c(t) = V(t)$, the collateralized exposure in such a model is exactly zero.

Assuming that $K(t) = c(t)$ is an idealization, which ignores the noninstantaneous nature of collateral settlement protocols and does not capture the fact that firms under stress may stop fully honoring margin calls, resulting in a divergence between the portfolio value and collateral value during some period $\delta$ prior to termination of the portfolio. In what we here call the classical model (see, for example, Pykhtin 2010), this particular lag effect is captured by modifying (2.1) to

$$E(t) = (V(t) - K(t))^+,$$

$$K(t) = c(t - \delta).$$

(3.1)

So, for instance, for a CSA with thresholds $h_B$ and $h_C$, from (2.2) we get

$$K(t) = (V(t - \delta) - h_C(t - \delta))^+ - (-V(t - \delta) - h_C(t - \delta))^+.$$

(3.2)

Having a mechanism for capturing divergence between collateral and portfolio value is an important improvement over the older model described above, and the classical model has gained widespread acceptance for both credit valuation adjustment (CVA) and regulatory capital calculations. Nevertheless, it hinges on a number of assumptions that are unrealistic. For instance, (3.1) assumes that both $B$ and $C$ will simultaneously stop paying margin at time $t - \delta$, freezing the margin level over the entire MPoR. In reality, if the party due to post collateral at $t - \delta$ happens to be the nondefaulting party $B$, it will often continue to post for some time even in the presence of news about the possible impending default of $C$. And, should $C$ miss a few margin payments (maybe under the guise of a dispute), $B$ would often continue to post collateral while it evaluates its options. This creates an asymmetry between posting and receiving collateral that the classical model fails to recognize.

In (3.1), the lag parameter $\delta$ is clearly critical: the larger $\delta$ is, the more $V(t)$ may pull from the frozen margin value at time $t - \delta$ and the bigger the expected exposure will become. In practice, $\delta$ is often determined in a fairly simplistic manner, eg, by using a fixed lag (commonly ten business days) or, more realistically, by adding a...
universal time delay to the remargining period of the CSA in question. This practice is echoed in regulatory guidelines, eg, in the Basel III Accord, where the MPoR is set equal to the remargining frequency minus one business day plus an MPoR floor that defaults to ten business days.\(^4\) With the high proportion of individually negotiated and amended features in real-life IMA/CSAs, using a “one size fits all” assumption may, however, lead to significant inaccuracies.

### 3.2 Assumptions about trade flows

Because large trade flows after the start of the MPoR may no longer be followed by collateral adjustment, they have the potential to either extinguish or exacerbate exposure. For this reason, the model assumptions with respect to the date when either party suspends trade flows are likely to have a significant effect on the counterparty credit loss. In one common interpretation of the classical model, it is simply assumed that both B and C will continue to pay all trade flows during the entire MPoR. As a consequence, the unpaid trade flow term UTF\(_t\) in (2.3) will be zero, consistent with (3.1). For ease of reference, we will denote this version of the classical model “Classical\(^+\)”.

In another, less common, version of the classical model, the assumption is that both B and C will stop paying trade flows at the moment the MPoR commences, ie, at time \(t - \delta\). In this case, we set the unpaid trade flows equal to\(^5\)

\[
UTF(t) = TF^{\text{net}}(t; (t - \delta, t)),
\]

where \(TF^{\text{net}}(t; (t', t''))\) is the time-\(t\) value of all net trade flows scheduled to be paid on the interval \((t', t'')\).\(^6\) We denote this version of the classical model “Classical\(^-\)”; it is associated with an exposure definition of

\[
E(t) = (V(t) + TF^{\text{net}}(t; (t - \delta, t]) - c(t - \delta))^+.
\]  

\(^3\) In practice, neither the Classical\(^+\) (3.1) or Classical\(^-\) (3.3) versions of the classical model are accurate representations of reality. Trade flows are likely paid, at least by B, in the beginning of the MPoR, and are likely not paid by at least C at the end of the MPoR. For instance, due to the CSA protocol for collateral calculations (see Section 4), there is typically at least a three-business-day lag between the start of the

---

\(^4\) The MPoR floor must be increased in certain cases, eg, for large netting sets, illiquid trades, illiquid collateral and recent collateral disputes; however, the increase is specified as a multiplier relative to the same default.

\(^5\) We measure time in discrete units of business days (bds), such that the notation \([u, s]\) is equivalent to \([u + 1 \text{ business day} , s]\).

\(^6\) If \(t\) is after the date of a margin flow, the trade flow value accrues forward from the payment date to \(t\) at a contractually specified rate.
Rethinking the margin period of risk

MPoR (the market observation date for the last full margin payment) and the date when B definitively observes that C has missed paying a margin flow; during this period, B would always make all trade payments unless C commits any additional contract violations. Even after B has determined that C has missed a margin payment, B’s nominal legal right to suspend payments following the breach would, as mentioned earlier, not always be exercised aggressively. Legal reviews, operational delays and grace periods can further delay the time when B would finally stop paying trade flows to C.

Another trade flow effect arises during the last two or three days of the MPoR (just prior to termination), where C has already defaulted and neither party is likely making trade payments. Here, the IMA stipulates that the missed trade flows in this period accrue forward at a contractually specified rate and become part of the bankruptcy claim. This gives rise to a termination period contribution to the UTF(t) term, in turn leading to an adjustment in the exposure.

4 FULL TIMELINE OF INTERNATIONAL MASTER AGREEMENT/CREDIT SUPPORT ANNEX EVENTS

Loosely speaking, the IMA concerns itself with the events of default, termination and closeout; and the CSA governs collateral exchanges, including the concrete rules for collateral amount calculations and posting frequencies. While we have touched on the workings of the IMA/CSA in previous sections, our model construction will require more detailed knowledge of certain provisions regarding the normal exchange of collateral, the legal options available in case of a missed payment and common bank policies with respect to availing itself of these options. A detailed exposition of the IMA and CSA legal complexities can be found in multiple sources, including on the ISDA website (www.isda.org); here, we provide only a brief summary to the extent necessary to develop our model. Our focus is on the development of a plausible timeline of events taking place around a default and subsequent portfolio termination.

4.1 Events prior to default

Let us assume that bank B is the Calculation Agent for computation of collateral amounts. As before, let $A_B$ and $A_C$ denote prescribed collateral amounts for B and C; as we discussed, these may differ from the collateral amounts $M_B$ and $M_C$ that are actually available if one of the parties fails to make a margin flow or changes the prescribed amount.

---

7 To ease comparison with actual contracts, in this section we capitalize official legal terms.
The following list describes the complete sequence of events taking place at times $T_0, T_1, \ldots$. We will simplify and condense them into a tractable model in the next section.

1. **Time $T_0$.** Our timeline begins at $T_0$, the “as-of” date at which the value of the portfolio and its collateral is measured, for use in the $T_1$ evaluation of the formulas for the Credit Support Amount (plainly, the amount of collateral). Typically, $T_0$ is the close of business on the business day before $T_1$.

2. **Time $T_1$.** We use $T_1$ to denote the last undisputed and respected Valuation Date prior to default. At time $T_1$,\(^8\) besides officially determining $A_B(T_0)$ and $A_C(T_0)$, bank B calculates the incremental payment amounts to itself and to C as $m_B = A_B(T_0) - M_B(T_0)$ and $m_C = A_C(T_0) - M_C(T_0)$, respectively. Taking into account any Minimum Transfer Amounts, the transfer amounts $m_B$ and $m_C$ should normally be communicated by B to C prior to a Notification Time (eg, 13:00 local time).

3. **Time $T_2$.** After receiving notice of the calculated collateral amount, C must initiate transfers of sufficient amounts of eligible collateral on the Payment Date $T_2$. Assuming that B managed to get the collateral amount notification sent to C prior to the Notification Time, $T_2$ defaults to one business day after $T_1$. If B is late in its notification, $T_2$ would be two business days after $T_1$. We here assume that the required amounts, which we recall were calculated at $T_1$ using market data from $T_0$, are all settled without incident at $T_2$. However, $T_2$ will be the last time that margin flows settle normally before the default takes place.

4. **Time $T_3$.** We let $T_3$ denote the next scheduled Valuation Date after $T_1$. If $\alpha$ is the average scheduled time between collateral calculations, we have approximately (ignoring business calendar effects) $T_3 \approx T_1 + \alpha$. At $T_3$ (hopefully before the Notification Time), B will send a payment notice to C, but C has now gotten into financial stress and will not be able (or willing) to pay further margin flows. Should C simply fail to pay collateral outright at the next Payment Date, a Credit Support Default could be triggered shortly thereafter (nonpayment of collateral is associated with a two-business-day grace period). To prevent this from happening, it is, as discussed earlier, likely that C would attempt to stall by disputing the result of the $T_3$ collateral calculation by B.\(^9\)

---

\(^8\) Note that while calculations are formally made on $T_1$, we use $T_0$ as the time argument on all margin amounts, to reflect the fact that the market data is observed at time $T_0$.

\(^9\) One of the authors still has vivid memories of how traders at Long-Term Capital Management suddenly started disputing even the most basic of swap pricing methodologies. Default followed shortly afterward.
(5) Time $\tau$. Exactly how long the margin dispute is allowed to proceed is largely a behavioral question that requires some knowledge of B’s credit policies and its willingness to risk legal disputes with C. Additionally, we need to consider the extent to which C is able to conceal its position of financial stress by using dispute tactics or, say, blaming operational issues on its inability to pay collateral. Ultimately, however, either B will conclude that C is in default on its margin flows (a Credit Support Default), or C will commit a serious contract violation such as failing to make a trade-related payment. At that point, B will conclude that a Potential Event of Default (PED) has occurred. We identify the time of this event as the true default time, $\tau$.

(6) Time $T_4$. Once the PED has taken place, B needs to formally communicate it to C, in writing. Taking into account mail/courier delays, legal reviews and other operational lags, it is likely that the communication time, denoted $T_4$, occurs slightly after the PED.

(7) Time $T_5$. After receipt of the PED notice, C will be granted a brief period of time to cure the PED. The length of this cure period is specified in the IMA and depends on both the type of PED and the specific IMA. For instance, if the PED in question is Failure to Pay, the default cure period (which may very well be overridden in the actual documents) is three business days in the 1992 IMA and one business day in the 2002 IMA. At the end of the cure period, here denoted $T_5$, an Event of Default (ED) formally crystallizes. We emphasize that, here, we do not associate $T_5$ (the “official” default time) with the true default time $\tau$; instead, we equate $\tau$ to the time of the actual event (the PED) that, after contractual formalities, leads to the default of C.

(8) Times $T_6$ and $T_7$. After the ED has taken place, B will inform C of the ED at time $T_6 > T_5$ and may, at time $T_7 > T_6$, elect to designate an Early Termination Date (ETD).

(9) Time $T_8$. The ETD is denoted $T_8$; per the IMA, it is required that $T_8 \in [T_7, T_7 + 20\text{ days}]$. The ETD constitutes the “as-of” date for the termination of C’s portfolio and collateral position. Many banks will aim for a speedy resolution in order to minimize market risk, and will therefore aim to set the ETD as early as possible. There are, however, cases for which this may not be optimal, as we discuss in Section 4.2.

(10) Subsequent events. Once the portfolio claim has been established as of the ETD, the value of any collateral and unpaid trade flows held by C is added to the amount owed to B. Paragraph 8 of the CSA then allows B to liquidate any securities collateral in its possession and to apply the proceeds against the
amount it is owed. Should the collateral be insufficient to cover what is owed to B, the residual amount will need to be submitted as a claim in C’s insolvency. The claim is then usually challenged by the insolvency representative and, where parties cannot agree, may be referred to court. It can sometimes take a long time before the claim is resolved by bankruptcy courts and the realized recovery becomes known. The interest on the recovery amount for this time is added to the amount awarded. Note that in this paper we focus exclusively on modeling the magnitude of the exposure and bankruptcy claim and do not challenge the established way of modeling the amount and timing of the eventual recovery using a loss-given-default (LGD) fraction.

The full timeline of IMA/CSA events is illustrated in Figure 1.

4.2 Some behavioral and legal aspects

While we have now established our event timeline, it remains for us to tie it to a proper model for exposure. In order to do so, we shall, as already mentioned, need to combine the timeline with coherent assumptions about bank and counterparty behavior in each subperiod. These assumptions should be determined not only by the rights available under IMA/CSA, but also by the degree of operational efficiency in serving notices and getting legal opinions, as well as by the levels of prudence injected into the
assumptions about the bank’s ability and willingness to strictly uphold contractual terms within each client group as it pertains to margin flows and disputes.

From a legal rights perspective, the most important observation is that once notice of a PED has been served (here: time $T_4$), the so-called suspension rights of the IMA (Section 2(a)(iii)) and the CSA (Paragraph 4(a)) allow B to suspend all trade- and collateral-related payments to C until the PED has been “cured”. The extent to which suspension rights are actually exercised, however, is quite situational. A particular danger is that B exercises its suspension rights due to a PED, but that subsequently the PED is ruled to be invalid. Should this happen, the bank can inadvertently commit a breach of contract, which, especially in the presence of cross-default provisions, can have serious consequences for the bank.

Another, somewhat counterintuitive, reason for B not to enforce its suspension rights is tied to the IMA’s Section 2(a)(iii), which can sometimes make it favorable for B to never designate an ETD. Indeed, if B owes C money, it would seem a reasonable course of action for B to simply

(a) never designate an ETD,

(b) suspend all payments on the portfolio until the default gets “cured”, which most likely will never happen.

This tactic basically allows B to walk away from its obligations on the portfolio when C defaults, effectively making B a windfall gain.

The strategy of delaying the ETD in perpetuity has been tested by UK courts and found to be legal.10 In the United States, however, local “safe haven” laws have been ruled to prevent ETDs of more than about one year. Still, a one-year delay may prove tempting if B has a big negative exposure to C and is unwilling to immediately fund the large cash outflow needed to settle. As most large banks are presumably unlikely to play legal games with the ETD, we shall not consider the topic further here, but just note that there is potentially room to make more aggressive model assumptions around ETDs than is done here.

5 SIMPLIFIED TIMELINE OF INTERNATIONAL MASTER AGREEMENT/CREDIT SUPPORT ANNEX EVENTS

It should be evident from the preceding section that the full timeline of IMA/CSA events reviewed in Section 4 is in many ways different to, and more complex than, what is assumed in both the Classical+ and Classical− versions of the classical model. However, it is equally evident that the full timeline is too complex to be modeled in every detail. In this section, we will offer a simplification of the timeline

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10 Contract language has been proposed by ISDA to prevent the issue.
designed to extract the events that are most important for exposure modeling. The resulting model offers several important improvements over the classical model while remaining practical and computationally feasible.

5.1 Identification of key time periods

To recap, recall first that the classical model considers only two dates in the timeline of default: the start and end of the MPoR. The start of the MPoR, denoted by $t - \delta$, is defined as the last observation date for which the margin was settled in full (a few days after the observation date). The end of the MPoR, denoted by $t$, is the observation date on which B’s claim is established. Note that $t$ coincides with the IMA’s Early Termination Date discussed in Section 4 (time $T_B$). An alternative name for ETD frequently used in counterparty credit risk modeling is “closeout date”.

In the classical model, there is no clear distinction between observation and payment dates, making it difficult to cleanly capture trade flow effects. For instance, in the Classical—version of the model, $t - \delta$ denotes both the last margin observation date and the date on which all trade flows cease. In reality, the last margin observation date is unlikely to be contentious and trigger stoppage of trade flows, as the margin payment to which the observation corresponds will only be missed by C several business days later. Specifically, if the market data is observed on day 0 and the valuation is performed on day 1, then only on day 2 (or day 3, if the notification was late) is the actual payment expected to be initiated. The length of this lag is of the same order of magnitude as typical assumptions for the length of the MPoR, and can be a source of considerable model error if not handled properly.

In the simplified timeline we propose here, we take care to keep track of the distinction between observation and payment dates, and also consider the possibility that B may stop a particular type of flow at a different time than C does. Accordingly, the model includes two (potentially different) observation dates for which B and C later settle their margin flows in full for the last time, and two (potentially different) dates when they pay their respective trade flows for the last time. The end of the MPoR is defined in the same way as in the classical model, to coincide with ETD. Table 1 summarizes the notation for these five dates in our simplified timeline.

The start of the MPoR in our model is $t - \delta$, which, in the notation of Table 1, may be defined symmetrically as $\delta = \max(\delta_B, \delta_C)$. We always expect the defaulting party C to stop posting margin no later than the nondefaulting party B. Therefore, we would very likely have $\delta_C \geq \delta_B$ and $\delta = \delta_C$.

The second column of Table 1 specifies which of the dates are observation dates, and which are settlement or payment dates. According to the notation established in Table 1, $\delta_B$ and $\delta_C$ are the lengths of time preceding the ETD during which changes in portfolio value no longer result in collateral payments by B and C, respectively.
TABLE 1  Notation for the dates in the simplified timeline.

<table>
<thead>
<tr>
<th>Event</th>
<th>Date type</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation date for the last margin flow by C</td>
<td>Observation</td>
<td>$t_C = t - \delta_C$</td>
</tr>
<tr>
<td>Observation date for the last margin flow by B</td>
<td>Observation</td>
<td>$t_B = t - \delta_B$</td>
</tr>
<tr>
<td>Date of last trade flow payment by C</td>
<td>Settlement</td>
<td>$t'_C = t - \delta'_C$</td>
</tr>
<tr>
<td>Date of last trade flow payment by B</td>
<td>Settlement</td>
<td>$t'_B = t - \delta'_B$</td>
</tr>
<tr>
<td>Early Termination Date</td>
<td>Observation</td>
<td>$t$</td>
</tr>
</tbody>
</table>

Similarly, $\delta'_B$ and $\delta'_C$ are the lengths of time preceding ETD during which the respective party does not pay trade flows. In, say, a “classical” ten-day MPoR model, we have $\delta_B = \delta_C = 10$ business days, with $\delta'_B = \delta'_C = 0$ for Classical+ and $\delta'_B = \delta'_C = 10$ business days for Classical−.

5.2 Establishing the sequence of events

A priori, the four events between the start and end of the MPoR in Table 1 can occur in any order. However, we will now explain why the table very likely shows the proper sequence of events.

As we discussed earlier, missing trade flows is often considered a more severe breach of contractual obligations than missing margin flows, especially as the latter may take the form of a margin valuation dispute. Therefore, it is reasonable to assume that neither party will stop paying trade flows before stopping the payment of margin flows. Accounting for the margin settlement lag between the observation date and the margin payment date, this yields

\[
\begin{align*}
\delta_C & \leq \delta_C - \text{margin settlement lag.} \\
\delta'_B & \leq \delta_B - \text{margin settlement lag.}
\end{align*}
\]  

(5.1)

It is also reasonable to assume that either of the two types of flows is missed first by the defaulting party C, and only then by the nondefaulting party B. This leads to the following additional constraints on the sequence of events within the timeline:

\[
\begin{align*}
\delta_C & \geq \delta_B \\
\delta'_C & \geq \delta'_B
\end{align*}
\]  

(5.2)

Except in rare and unique situations (such as outright operational failures), B would not continue to pay margin flows once C commits a more serious violation by missing a trade flow, resulting in

\[
\delta'_C \leq \delta_B - \text{margin settlement lag.}
\]  

(5.3)
Combining these inequalities results in the chronological order of events shown in Table 1.

5.3 Evaluation of survival probability

As was the case for the classical model, our setup anchors the exposure date \( t \) at the termination date (ETD), at the very end of the MPoR. The ETD is the same for both parties, and constitutes a convenient reference point for aligning the actions of both parties. We emphasize that the ETD for which exposure is evaluated does not coincide with the date at which survival probability is evaluated, eg, for the computation of CVA. In our simplified timeline, the counterparty survival probability should be evaluated for \( t - \delta_C \), the last date when \( C \) stops paying trade flows. Hence, if \( EE(t) \) is the expected exposure anchored at the ETD \( t \), then the incremental contribution to (unilateral) CVA from time \( t \) is, under suitable assumptions, \( EE(t) \, d\mathbb{P}(t - \delta_C) \), where \( \mathbb{P} \) is the survival probability under the model’s measure; see Section 7.3.3 for concrete examples.

Evaluating the default probability at the anchor date \( t \) rather than \( t - \delta_C \) will introduce a slight error in computing the survival probability. While this error is relatively small and is often ignored by practitioners, it takes virtually no effort, and has no impact on model efficiency, to evaluate the survival probability at the right date.

6 TIMELINE CALIBRATION

As we mentioned earlier, the specific IMA/CSA terms for a given counterparty should ideally always be examined in detail, so that any nonstandard provisions may be analyzed in terms of their impact on the timeline. For those cases when such a bespoke timeline construction is not practical (typically for operational reasons), we will here propose two standard (“reference”) parameterizations of our timeline. This will also allow us to demonstrate the thought processes behind timeline calibration, and will provide some useful base cases for our later numerical tests.

While factors such as portfolio size and dispute history with the counterparty should, of course, be considered in establishing the MPoR, we believe that an equally important consideration in calibrating the model is the nature of the expected response by \( B \) to missed margin or trade flows by \( C \). Even under plain vanilla IMA/CSA terms, experience shows that reactions to contract breaches are subject to both human and institutional idiosyncrasies, rendering the MPoR potentially quite variable. Recognizing that one size does not fit all, we shall therefore consider two different calibrations: one “aggressive”, which assumes a best-case scenario for rapidly recognizing the

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11 Effectively, we assume that default is due to failure-to-pay.
impending default and taking swift action; and one “conservative”, which takes into account not only a likely delay in recognizing that counterparty default is imminent, but also the possibility that the bank may not aggressively enforce its legal rights afforded under IMA and CSA in order to avoid damaging its reputation. In both scenarios, we also assume daily remargining: if a CSA calls for less frequent margin calls than this, the MPoR must be lengthened accordingly.

6.1 Aggressive calibration

The Aggressive calibration applies to the trading relationship between two counterparties that both have strong operational competence, and where there is little reputational risk associated with swift and aggressive enforcement of the nondefaulting party’s legal rights against the defaulting party. A good example would be trading between two large dealers, both willing to aggressively defend against a possible credit loss. We would here assume that credit officers are diligent in their monitoring of the counterparty and generally able to see a default developing, rather than being caught by surprise.

Under Aggressive calibration, the event of C missing (or disputing) a margin call by any nontrivial amount will, given C’s sophistication, immediately alert B that an impending default is likely. B will not be misled by claims of a valuation dispute or other excuses, and will send notice of a Credit Support Default under the IMA/CSA the next business day after the breach of the margin agreement. At the same time, to further protect itself, B will stop both margin flows and trade flows. The counterparty C is assumed to simultaneously stop paying margin and trade flows as well, so that no further payments of any kind are exchanged by the parties.

The simultaneous action by both parties in our Aggressive scenario to stop paying trade flows at the earliest possible moment results in the elimination of all settlement risk: the possibility that the bank may continue paying on its trade flow obligations while not receiving promised payments in return. In the context of cross-currency trades, this type of settlement risk is frequently referred to as “Herstatt” risk, after the bank that famously caused large counterparty losses in this manner (see, for example, https://en.wikipedia.org/wiki/Settlement_risk). Such risk will be captured in our Conservative calibration case below and discussed in more detail in Section 7.2.

Despite B’s immediate (and rather aggressive) response, the MPoR will still be fairly long due to the way the IMA/CSA operates in practice. In particular, note that the first period in our simplified timeline is between the last observation date for which margin was fully settled and the first date on which C misses a margin flow. As it takes (at least) two business days to settle a margin payment, and it takes an additional one business day to settle the last margin successfully and make the first margin payment that was not settled successfully, a minimum of three business days

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will always accrue from the start of the MPoR to a margin-related PED. Further, once the margin flow is missed, C must send two notices and permit a grace period (usually two business days) to cure the violation before an ED has officially taken place and an ETD has been designated. Since an ETD cannot be designated prior to the default event, it is unlikely that the MPoR can ever be less than seven business days. It is remarkable that, even under the most aggressive set of assumptions, MPoR is still only three business days shorter than the classical two-week MPoR.

The detailed taxonomy of our Aggressive timeline is listed in Table 2, and essentially splits the MPoR into two sections: a margin delay period of three business days, and a default resolution period of four business days. During the latter period, B and C cease paying on the first day, leaving a period of three business days where neither party makes any payments. Note that we assume that the ETD is declared to coincide with the ED, ie, the bank will terminate as quickly as legally possible.

### 6.2 Conservative calibration

The Conservative calibration is intended to cover the situation when the bank’s enforcement of its rights under the IMA/CSA is deliberate and cautious, rather than swift. There may be several reasons for such a situation, sometimes acting together.

First, a bank, if overly trigger-happy, can gain a market-wide reputation as being rigid and litigious, potentially causing clients to seek other trading partners. In fact, should aggressive legal maneuvers be applied to counterparties that may be considered “unsophisticated”, there is even the potential for the bank to be perceived as predatory by the larger public. Second, there are situations where exercising legal rights would cause an unattractive leak of information into the broader market. As mentioned, this may, for instance, happen if the formal collateral dispute methodology in Paragraph 5 of the ISDA CSA is activated: the market poll mechanism inherent in the methodology would inevitably reveal the positions held with the counterparty to competing banks. Third, sometimes an aggressive interpretation of legal rights can backfire in the form of lawsuits and countermeasures by the counterparty. For example, even when a bank may have the right to withhold payments (eg, under Section 2(a)(iii)), it would often elect not to exercise this right immediately, out of concern that a counter-ED would be raised against it or that withholding payments would exacerbate the liquidity situation of the counterparty, potentially exposing the bank to liability and lawsuits. As mentioned, a particular danger is that the bank exercises its suspension rights due to a PED, but that subsequently the PED is ruled not to be valid. Should this happen, the bank can inadvertently commit a breach of contract.

Of course, even if a bank may potentially be willing to aggressively exercise its rights, it might not have the operational capacity to do so quickly. For example, the bank may not be able to perform the requisite legal review at short notice, or it may
not have the efficiency needed to always get notices mailed out at the earliest possible date. On top of this, there is always the potential for human or technology-related errors and oversights.

While it is harder to get concrete data to estimate a reasonable timeline for the Conservative case (this case being dependent on not only IMA/CSA details but also the specific banks’ reputational considerations), under a perfectly reasonable set of assumptions the MPoR ends up being more than twice as long as for our Aggressive case above. Under this calibration choice, the Conservative scenario assumes that the sum of margin dispute negotiations, operational delays, human errors, legal reviews, etc, is eight business days, yielding a total MPoR of fifteen business days. One plausible scenario with daily remargining could be the following.

$t - 15$: B observes the portfolio value as needed for margin transfer amount #1 as of $t - 15$.

$t - 14$: B sends margin call #1 to C; B observes margin transfer amount #2.

$t - 13$: B sends margin call #2; C honors margin call #1; B observes margin transfer amount #3.

$t - 12$: C fails to honor margin call #2 and initiates dispute; B tries to resolve the dispute while still paying and calculating margin.

$t - 7$: C fails to make a trade payment.

$t - 6$: B stops paying margin and sends the PED notice.

$t - 5$: C receives PED; B keeps making trade payments.

$t - 3$: The PED is not cured.

$t - 2$: B stops trade payments and sends ED notice to C, designating $t$ as the ETD.

$t$: ETD.

Note that we have a number of different margin transfers (denoted #1, #2 and #3) active simultaneously, reflecting the interlacing nature of daily margin calls. Note also that, unlike the earlier Aggressive calibration, the above scenario explicitly involves settlement risk, as a time period exists in which only B pays trade flows (from $t - 7$ to $t - 3$, both dates inclusive).

To translate the scenario above into the notation of Section 5, first note that $\delta_C = 15$, as the observation date of the last margin call (#1) honored by C is $t - 15$. Second, as B makes its last possible margin payment at time $t - 7$, based on an observation at time $t - 9$, we have $\delta_B = 9$. Third, as C fails to make a trade payment at $t - 7$, C’s last payment date is $t - 8$, and therefore $\delta_C' = 8$. And, finally, since B stops its trade payments at $t - 2$, we must have $\delta_B' = 3$. 

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TABLE 2 MPoR periods for CSAs with daily remargining.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conservative</th>
<th>Aggressive</th>
<th>Classical+</th>
<th>Classical−</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_C$</td>
<td>15</td>
<td>7</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\delta_B$</td>
<td>9</td>
<td>6</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\delta'_C$</td>
<td>8</td>
<td>4</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>$\delta'_B$</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

All values are given in business days.

6.3 Summary and comparison of timelines

Our Aggressive and Conservative scenarios are presented in Table 2 in the notation of Section 5. For reference, the Classical+ and Classical− versions of the classical model (see Section 3) are also presented in the table. Note that the ten-business-day assumption of the classical MPoR model lies between the two calibration choices we propose and is closer to the Aggressive scenario.

Our Aggressive and Conservative parameter choices represent two opposite types of bank–counterparty relationship; they may also be used as two limit scenarios for materiality and model risk analysis. Of course, the best approach would always be to set model parameters based on prudent analysis of the firm’s historical default resolution timelines, to the extent that this is practicably feasible.

We should note that there could conceivably be a stochastic element to a firm’s classification as Aggressive or Conservative, as, say, time lags could shrink in those scenarios where exposures are particularly high, since a firm may act with greater urgency and determination in such circumstances. More broadly, all the time lags in our model could in principle be treated as random variables to be simulated as part of exposure computations, to reflect random operational events or firm behavior. The parameterization of such a setup would obviously be fairly challenging, and it is debatable whether increasing the number of model parameters in this way is warranted in practice.

7 ADVANCED MODEL FOR COLLATERALIZED EXPOSURE

To formulate our model in more precise mathematical terms, let us return to (2.3) and consider how to draw on our analysis in Sections 4–6 to reasonably specify both the collateral amount $K(t)$ and the value $\text{UTF}(t)$ of net unpaid trade flows.

7.1 Unpaid margin flows and margin flow gap

As in the classical model, we assume that the MPoR starts at time $t_C = t - \delta_C$, the portfolio observation date associated with the last regular collateral posting by
Rethinking the margin period of risk

C. Recall that the classical model further assumes that B will stop posting collateral simultaneously with C, so that (cf. (3.1))

\[ K(t) = c(t_C), \quad (7.1) \]

where \( c(t_C) \) denotes the CSA-prescribed collateral support amount calculated from market data observed at time \( t_C \).

In contrast to (7.1), our model assumes that B will continue posting and returning collateral to C for all contractual margin observation dates \( t_i \), whenever required by the CSA, after \( t_C = t - \delta_C \) and up to and including the observation date \( t_B = t - \delta_B \). We will refer to the presence of an observation period of nonzero length, during which B posts and returns collateral but C does not, as the “margin flow gap”.

Here, we always expect \( t_B \geq t_C \); therefore, in effect, we assume the possibility of a time interval \( (t_C, t_B) \) in which only B honors its margin requirements. In this interval, B can match contractually stipulated amounts \( c(t_i) \) only when they involve transfers from B to C. This asymmetry results in B holding at time \( t \) the smallest collateral computed on the observation interval \([t_C, t_B]\), ie,

\[ K(t) = \min_{t_i \in [t_C, t_B]} c(t_i). \quad (7.2) \]

The “worst-case” form of (7.2) clearly provides a less optimistic view on available collateral than the classical model, resulting in larger exposure whenever there are multiple collateral observation dates in \([t_C, t_B]\).\(^{12}\) All other things being equal, the difference in exposure relative to the classical model will increase with \( \delta_C - \delta_B \). If \( \delta_C - \delta_B \) is kept constant, the difference will increase with more frequent remargining. Note that (7.2) equals (7.1) when \( \delta_B = \delta_C \).

7.2 Unpaid trade flows and trade flow gap

According to our assumptions in Section 5, the last date when C is still paying trade flows is \( t'_C = t - \delta'_C \), and the last date when B is still paying trade flows is \( t'_B = t - \delta'_B \geq t'_C \). We shall refer to the period when B is still paying trade flows while C does not as the “trade flow gap”.

The value of the net trade flows unpaid by the termination date \( t \) can be expressed using the notation of Section 3.2 as

\[ \text{UTF}(t) = \text{TF}^{C \to B}(t; (t'_C, t)) + \text{TF}^{B \to C}(t; (t'_B, t)) \]

\[ = \text{TF}^{C \to B}(t; (t'_C, t'_B)) + \text{TF}^{\text{net}}(t; (t'_B, t)), \quad (7.3) \]

where an arrow indicates the direction of the trade flows, and C \( \to \) B (B \( \to \) C) trade flows have a positive (negative) sign.

\(^{12}\) It is assumed that one of the observation dates \( t_i \) always coincides with the start of the MPoR, \( t_C \).
In calculating (7.3), care needs to be taken in how trade flows are aggregated and accrued to the termination date \( t \).

- Cashflows of opposite directions scheduled to be paid in the same currency on the same date (for instance, the two legs of an ordinary single-currency interest rate swap) in period \( (t'_B, t] \) are aggregated (netted) at the cashflow date. Therefore, only the aggregated amount (their difference) enters into (7.3). The aggregated amount of missed cashflows should be accrued to time \( t \) at the interest rate of the currency in question, and then converted to B’s domestic currency.

- Cashflows in opposite directions scheduled to be paid in different currencies on the same date (for instance, the two legs of a cross-currency (XCCY) interest rate swap) in period \( [t'_B, t] \) are not netted at the cashflow date. The missed cashflow amounts in each currency should be accrued to time \( t \) at the relevant interest rates, and then converted to B’s domestic currency.

- The value of each asset flow (for instance, a swap that would result from exercising a physically settled swaption) should be obtained through pricing the undelivered asset in B’s domestic currency at time \( t \). Generally, asset flows are not aggregated.

To analyze the impact of the assumptions in (7.3), let us for simplicity consider a zero-threshold margin agreement with no MTA/rounding. Then, from (7.2), the collateral available to B at the termination date can be written as

\[
K(t) = V(t_{\text{col}}), \quad t_{\text{col}} \triangleq \arg \min_{t_i \in [t_C, t_B]} V(t_i).
\]  

(7.4)

Substituting (7.3) and (7.4) into (2.3) for the simple CSA considered yields

\[
E(t) = [V(t) - V(t_{\text{col}}) + TF_{C\rightarrow B}(t; (t'_C, t'_B)) + TF_{\text{net}}(t; (t'_B, t))]^+. \quad (7.5)
\]

Equation (7.5) implies that trading flows from B to C can occur within the MPoR. These trade flows have the potential to generate large spikes in the exposure profile, especially in the presence of a trade flow gap, when only B pays trade flows. To see this, we drill down further into the exposure components of (7.5) as follows. First, ignoring minor discounting effects inside the MPoR, let us represent the portfolio value at time \( t_{\text{col}} \) as the sum of the portfolio’s forward value \( V^F \) to time \( t \) and the value of all the trade flows taking place after \( t_{\text{col}} \) and up to and including \( t \):

\[
V(t_{\text{col}}) = V^F(t_{\text{col}}; t) + TF_{\text{net}}^+(t_{\text{col}}; (t_{\text{col}}, t)). \quad (7.6)
\]
Here, we may further write

\[ \text{TF}^{\text{net}}(t_{\text{col}}; (t_{\text{col}}, t)] = \text{TF}^{\text{net}}(t_{\text{col}}; (t_{\text{col}}, t_{B}')] \\
+ \text{TF}^{C\rightarrow B}(t_{\text{col}}; (t_{C}', t_{B}']) \\
+ \text{TF}^{B\rightarrow C}(t_{\text{col}}; (t_{C}', t_{B}']) \\
+ \text{TF}^{\text{net}}(t_{\text{col}}; (t_{B}', t]), \]

which, together with (7.6), allows us to restate (7.5) in the following illuminating form:

\[
E(t) = [V(t) - V^F(t_{\text{col}}; t) \\
+ \text{TF}^{C\rightarrow B}(t; (t_{C}', t_{B}')] - \text{TF}^{C\rightarrow B}(t_{\text{col}}; (t_{C}', t_{B}')) \\
+ \text{TF}^{\text{net}}(t; (t_{B}', t)) - \text{TF}^{\text{net}}(t_{\text{col}}; (t_{B}', t)] \\
- \text{TF}^{\text{net}}(t_{\text{col}}; (t_{C}', t_{B}']) \\
- \text{TF}^{B\rightarrow C}(t_{\text{col}}; (t_{C}', t_{B}'))]^+. \tag{7.7}
\]

Here, we have arranged the terms in (7.5) on five separate lines, corresponding to the following five contributions to exposure.

1. The change of the portfolio forward value to time \( t \), driven by the change in market factors between \( t_{\text{col}} \) and \( t \). This term is driven by the volatility of market factors between \( t_{\text{col}} \) and \( t \); it produces no spikes in the expected exposure profile.

2. The change of the value of the trade flows scheduled to be paid (but actually unpaid) by \( C \) in the interval \( (t_{B}', t] \), resulting from the change of market factors between \( t_{\text{col}} \) and \( t \). This term is driven by the volatility of market factors between \( t_{\text{col}} \) and \( t \); it produces no spikes in the expected exposure profile.

3. The change in the value of the net trade flows between \( C \) and \( B \) scheduled to be paid (but actually unpaid) in the interval \( (t_{B}', t] \), resulting from the change of market factors between \( t_{\text{col}} \) and \( t \); this term likewise produces no spikes in the expected exposure profile.

4. The negative value of the net trade flows between \( C \) and \( B \) scheduled to be paid (and actually paid) in the interval \( (t_{\text{col}}, t_{C}'] \). Paths where \( B \) is the net payer (so that \( TF \) is negative) contribute to upward spikes in the EE profile.

5. The negative value of the trade flows scheduled to be paid (and actually paid) by \( B \) to \( C \) in the interval \( (t_{C}', t_{B}'] \). Whenever such trade flows are present, \( B \) is always the payer, leading to upward spikes in the EE profile. Further, in some cases, the spikes arising from this term can be extreme, eg, a scheduled notional exchange in a cross-currency swap when \( B \) pays the full notional but receives nothing.
7.3 Numerical examples

To gain intuition for our model, we now present exposure profiles and CVA metrics for several trade and portfolio examples, using both Aggressive and Conservative calibrations. We focus on ordinary and cross-currency swaps, as these instruments are the primary sources of exposure in most banks. For all numerical examples, we drive the stochastic yield curve scenarios with a one-factor Hull–White model; for our cross-currency swap examples, the foreign exchange rate is assumed to follow a Black–Scholes model.\(^\text{13}\)

7.3.1 Individual swap results

First, let us examine how our model differs from the classical exposure approach. In Figure 2 we have used Monte Carlo simulation on a US$10 million one-year par-valued vanilla interest rate swap to compare exposures of our Conservative calibration with those computed from the Classical+ and Classical− models (see Table 2). To make our comparisons meaningful, we have overridden the default setting of ten business days for the Classical method, and instead set it equal to fifteen business days, the length of the MPoR for our Conservative calibration.

\(^\text{13}\) The source code for the numerical examples in this paper is available at http://modval.org/models/mpr/ and http://modval.org/papers/aps2016/.
In Figure 2, note that the Classical— calibration is, of course, the least conservative setting, as it ignores the effects of both trade flows and margin asymmetry. The Classical+ calibration tracks the Classical— results at most times, but it contains notable spikes around the last three cashflow dates. The spikes originate from scenarios where B pays a net positive coupon on a cashflow date, yet, by assumption, receives no margin in return. The spikes commence whenever the exposure date (the ETD in our convention) hits a coupon payment date, and they cease fifteen business days later, the length of the MPoR. Our Conservative calibration results also contain spikes around cashflow dates, although these differ from those of the Classical+ calibration in several ways. First, the Conservative calibration correctly recognizes that there will always be an interval toward the end of the MPoR (after time $t_B'$) where B and C will have both stopped paying margin and coupons; as a result, the spikes of the Conservative calibration start later (here, three business days later) than those of the Classical+ calibration. Second, the initial part of the spike (in the period from $t_C'$ to $t_B'$) is substantially higher for the Conservative calibration, due to the assumption of only B paying cashflows in this subperiod. The remainder of the spike is comparable in height to the Classical+ spike.

Between spikes, we note that our Conservative calibration produces higher exposures than both the Classical+ and Classical— methods, by around 40%. This is, of course, a consequence of the “worst-case” margin asymmetry mechanism in (7.2), the effect of which will grow with the diffusion volatility of the rate process. Note that the last coupon period has no exposure between spikes, since the volatility of the swap price vanishes after the last floating rate fixing at 0.75 years.

The comparison of the Aggressive calibration with the Classical+ and Classical— calibrations is qualitatively similar to the results in Figure 2, so we skip a detailed comparison and just note that the pick-up in exposure from margin asymmetry falls to about 15%, rather than the 40% we observed for the Conservative calibration: a result of the fact that the “worst-case” margin result is established over much fewer days for the Aggressive calibration. A comparison of exposure profiles for the Aggressive and Conservative calibrations can be found in Figure 3; as expected, the Conservative calibration leads to both bigger and wider exposure spikes as well as higher exposure levels between spikes.

While instructive, we note that our one-year vanilla swap example is quite benign exposure-wise: not only is the instrument very short-dated, it also permits netting of coupons on trade flow dates, thereby reducing the effects of trade flow spikes. We can relax both effects by increasing the maturity of the swap, and by making the fixed

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14 Here, no spike occurs on the first quarterly cashflow date, as we assume that the floating rate is fixed at 2%, making the net cashflow zero in all scenarios.
FIGURE 3 Expected exposure for one-year vanilla swap.

US$10 million one-year payer swap, 2% coupon; fixed and floating coupons paid quarterly. Simulation and exposure setup as in Figure 2.

FIGURE 4 Expected exposure for ten-year vanilla swap.

US$10 million ten-year payer swap, 2% coupon; fixed coupons paid semiannually, floating coupons paid quarterly. Model and exposure setup otherwise as in Figure 2.

and floating legs pay on different schedules. The exposure results for such a case are shown in Figure 4.

In Figure 4, upward exposure spikes occur twice per year, whenever the bank must make a (semiannual) fixed payment. On dates when the counterparty makes a (quarterly) floating rate payment that is not accompanied by a fixed payment by
the bank (also twice per year), a narrow downward spike now emerges, due to the schedule delay in transferring back the coupon to the counterparty through the margin mechanism. Note that the exposure between spikes is much larger than in Figure 3: a consequence of the higher volatility of a ten-year swap compared with a one-year swap. Of course, as the swap nears maturity, its duration and volatility die out, so the nonspike exposure profile predictably gets pulled to zero at the ten-year horizon. Also as predicted, the Aggressive calibration produces much lower exposures than the Conservative calibration, by nearly a factor of two.

More extreme trade flow spikes will occur for cross-currency swaps, where neither coupons nor the final notional payment can be netted. The notional payment, in particular, can induce a very significant exposure spike (Herstatt risk), whenever the exposure model allows a trade flow gap. As we recall, our Conservative calibration has a trade flow gap, but our Aggressive calibration does not. As Figure 5(b) confirms, the exposure for the Conservative calibration has a very large terminal spike that is not present in the Aggressive calibration. Like the Conservative calibration, the Aggressive calibration will, of course, still produce exposure spikes at cashflow dates, due to margin effects (see Figure 5(a)). For variation and for realism, note that in Figure 5 we have elected to use a seasoned deal traded at a time when the interest rate levels and the EUR/USD foreign exchange (FX) rate were all higher than at present. As a consequence, the principal exchange is likely far from breakeven, resulting in a large exposure spike at maturity. Although it is smaller than for the Conservative calibration, note that a spike at maturity is also present for the Aggressive calibration: while both B and C pay the principal exchange, C does not make the margin transfer for the balance of the principal payments.

7.3.2 Portfolio results

For individual trades, the presence of localized spikes in the exposure profiles may ultimately have a relatively modest impact on credit risk metrics, such as the CVA; after all, the likelihood of a counterparty default in a narrow time interval around a quarterly or semiannual cashflow event is typically low. For a portfolio of swaps, however, the spikes will add up and can affect the net exposure profile nearly everywhere. To show this, we picked fifty interest rate swaps with quarterly floating rate payments and semiannual fixed rate payments of 2%. The terms of the swaps were randomized as follows:

- notionals were sampled uniformly on the interval from 0 to US$10 million;
- the direction of fixed-leg payments (payer or receiver) was random;
- the start date of each swap was subject to a random offset to avoid complete MPoR overlaps;
- the swap maturities were sampled uniformly over the ten-year interval.
FIGURE 5 Expected exposure for ten-year cross-currency swap.

(a) Excluding exposure at maturity. (b) Including exposure at maturity. US$10 million ten-year USD/EUR cross-currency payer swap, fixed 3% euro semiannual coupon against a quarterly US dollar floating rate with US$15 million notional and €10 million notional. The margin agreement uses daily margin transfers, no thresholds and no MTA/rounding. Expected exposures are computed by brute-force daily simulation (5000 paths), with default exposures captured in Aggressive and Conservative calibrations, as specified on the graphs. Scenarios computed by uncorrelated Hull–White models for the interest rates, with the initial US dollar and euro yield curves flat at 2% and 1%, respectively, Gaussian (basis point) volatilities of 1% and mean reversion speeds of 5%. The exchange rate is lognormal, with USD/EUR spot at 1.2 and a constant FX diffusion volatility of 10% (as the foreign exchange (FX) rate has random drift (due to the volatility of the US dollar and euro interest rates), the implied FX volatility is, of course, larger than the diffusion volatility). For clarity, part (a) excludes the final spike in order to show more detail in the prematurity profile; the spike is shown in (b).
A portfolio of fifty interest rate swaps, as defined in the text; fixed coupons (2%) paid semiannually; and floating coupons paid quarterly. Simulation and exposure setup are otherwise as in Figure 2, except 1000 paths were used for all portfolio examples.

The resulting expected exposure profiles are shown in Figure 6. Note how both the Conservative profile and (to a lesser extent) the Aggressive profile include frequent spikes around trade flow times above the “baseline” exposure level. As we shall see in Section 7.3.3, these spikes make a significant contribution to CVA metrics. As before, the exposure under Conservative calibration is about twice that under the Aggressive calibration.

To repeat the portfolio results with cross-currency swaps, we constructed a fifty-deal portfolio by randomization, using the following rules:

- euro notionals are sampled uniformly on the interval from 0 to US$10 million;
- US dollar notionals are 1.5 times euro notionals;
- the euro leg has a fixed semiannual coupon of 3%; the US dollar leg has a floating quarterly coupon;
- the direction of fixed-leg payments (payer or receiver) is random;
- the start date of each swap is subject to a random offset to avoid complete MPoR overlaps;
- swap maturities are sampled uniformly on the interval from one to ten years.

As for Figure 5, we generated the swaps within the portfolio such that the principal exchange and fixed coupon are not at-the-money, to mimic the typical situation for a
A portfolio of fifty cross-currency rate swaps, as defined in the text; fixed euro coupons (3%) paid semiannually; and floating US dollar coupons paid quarterly. Simulation and exposure setup is otherwise as in Figure 5, except 1000 paths were used for all portfolio examples.

portfolio of seasoned trades. As shown in Figure 7, the exposure for the Conservative calibration is, as expected, here dominated by a series of Herstatt risk spikes, one per swap in the portfolio. As before, we generated the swaps within the portfolio such that the principal exchange and fixed coupon are not at-the-money, to mimic the typical situation for a portfolio of seasoned trades done at the time of different market rates.

7.3.3 CVA results

As mentioned earlier, a common use of expected exposure results is the computation of CVA. Under suitable assumptions, B’s unilateral CVA may be computed from the expected exposure (EE) profile as

$$\text{CVA} = (1 - R) \int_0^\infty P(u + \delta_C^C) \text{EE}(u + \delta_C^C) dX(u), \quad (7.8)$$

where $R$ is the recovery rate, $P(t)$ is the time-0 discount factor to time $t$ and $X(t)$ is the time-0 survival probability of $C$ to time $t$. Note that, as discussed in Section 5.3, we offset the exposure profile by $\delta_C^C$ to properly align exposures with default events.

The CVA metric serves as a convenient condensation of the exposure profiles in Sections 7.3.1 and 7.3.2 into single numbers, and Table 3 lists CVA results for the instruments/ portfolios in Figures 4–7. We discretized the integral in (7.8) on a daily grid, assumed $R = 40\%$ and let the forward default intensity be constant at $2.5\%$ such that $X(t) = \exp(-0.025t)$. Note that, for reference, Table 3 also includes results...
for the Classical method, with MPoR lengths equal to both the Aggressive calibration (seven business days) and the Conservative calibration (fifteen business days).

The results in Table 3 confirm what we saw earlier. For instance, the CVA for the Aggressive calibration is 50–70% smaller than CVA for the Conservative calibration. In addition, the CVA of the Conservative calibration is 50–100% larger than that of the Classical+ calibration (at similar MPoR), which in turn is larger than the CVA of the Classical− calibration by around 5–25%. Not surprisingly, the CVA results for the XCCY portfolio are particularly high in the Conservative calibration, due to Herstatt risk.

### 8 IMPROVEMENT IN COMPUTATION TIMES

In exposure calculations for realistic portfolios, horizons can be very long, often exceeding thirty years. For such lengthy horizons, brute-force Monte Carlo simulation of exposures on a daily, or even weekly, time grid will often be prohibitively slow. It is therefore common to use daily simulation steps only for the earliest parts of the exposure profile (eg, the first month), and then gradually increase the time step length over time to monthly or quarterly, in order to keep the total number of simulation dates manageable. Unfortunately, such a coarsening of the time grid will inevitably
fail to capture both the “worst-case” margin effect and the trade spikes that are key to our exposure model.

In this section, we discuss ways to capture exposure without having to resolve to brute-force daily simulation. We first review a common speed-up technique for the Classical model (the coarse grid lookback method), highlighting its shortcomings and pitfalls. We then propose an improved practical technique based on a Brownian bridge.

8.1 The coarse grid lookback method and its shortcomings

Assume that portfolio simulation is done not daily, but instead only on a coarse grid \( \{s_j\} \), where \( j \) runs from 1 to \( J \). We use \( s \) rather than \( t \) to distinguish the model grid from the daily margin calculation grid.

In the classical model, the collateral depends only on the portfolio value at the start and at the end of the MPoR, i.e., \( s_j - \delta \) and \( s_j \) (see (3.1) and (3.2)), where the MPoR is usually around \( \delta = 10 \) business days for CSAs with daily remargining. To achieve acceptable computational performance, the time step of the coarse model grid, \( s_j - s_{j-1} \), must be significantly greater than the length of the MPoR. This, however, would preclude the portfolio value from being established at \( s_j - \delta \). The coarse grid lookback method deals with this issue by simply adding a second “lookback” time point \( s_j - \delta \) to all “primary” measurement times \( s_j \), in effect replacing each node of the coarse model grid by a pair of closely spaced nodes. For each simulated portfolio path, the portfolio value at a lookback point is then used to determine the collateral available at the corresponding primary time point.

The coarse grid lookback scheme causes, at worst, a factor of \( \times 2 \) slowdown relative to valuing the portfolio once per node of the coarse model grid. If even a \( \times 2 \) performance loss is not acceptable, a Brownian bridge constructed between the primary coarse grid nodes can be used to interpolate the value of the portfolio at each lookback point (see, for example, Pykhtin 2009). Note that the use of the Brownian bridge for this purpose should not be confused with its use in Section 8.2.

The coarse grid lookback method is a common way of addressing the mismatch between the long time step of the coarse model grid and the much shorter MPoR. Similarly to commonly used models of uncollateralized exposure, the method produces accurate (with respect to the underlying assumptions of the Classical model) exposure numbers at the coarse time grid points, but provides no information on exposure between the grid points. For uncollateralized positions, the exposure profiles are reasonably smooth, so we can safely interpolate between the grid points when calculating integral quantities, such as CVA. In the collateralized case, however, we cannot rely on such an interpolation because the true exposure profile, as we have seen, is likely to have spikes and jumps between the grid points. The coarse grid lookback
method has no means of determining the position or magnitude of these irregularities between the grid points, and thus it is not suitable for CVA or capital calculations.

To briefly expand on this, consider the Classical+ version of the classical model. Here, it is assumed that all trade flows are paid within the MPoR, where, as we showed in Section 7, trade flows often result in exposure spikes. An exposure profile computed with daily time steps would consequently show spikes from all trade flows until the maturity of the portfolio. In contrast, in a typical implementation with sparsely spaced MPoRs, only trade flows that happen to be within one of the actually simulated MPoRs may result in spikes; the exposure profile would then miss the spikes related to all other trade flows. Further, as the location of simulation points will likely change with the advancement of calendar time, trade flows would move in and out of the simulated MPoRs, and the exposure profile spikes that one will report on any given day may very well differ significantly from those that were reported the day before. This, in turn, causes CVA or risk capital to exhibit significant, and entirely spurious, oscillations.

While the Classical—exposure model does not exhibit outright spikes, its exposure profiles still exhibit jumps around significant trade flows (see Figure 2). The classical coarse-grained implementation would not be able to resolve the position of these jumps, instead only showing the cumulative jump between two adjacent exposure measurement points often separated by many months. This creates another source of instability, present in both Classical— and Classical+ versions of the classical model.

To illustrate the effects described above, let us define the concept of time-“forward” CVA, denoted $CVA^t$, obtained by

(i) changing the lower integration limit in (7.8) from 0 to $t$,

(ii) dividing the result by $P(t)X(t)$.

Using the same portfolio of fifty EUR/USD cross-currency swaps as in Figure 7, Figure 8 shows the $t$-dependence of $CVA^t$ on a daily grid up to portfolio maturity.

As CVA is an integral of exposure, spikes in exposure profile should result in jumps rather than oscillations in $CVA^t$. However, when one of the coarse grid look-back method’s sparsely located “MPoR windows” moves over a large trade flow, the contribution to CVA temporarily increases, only to drop back when the window moves past the large trade flow. This results in the large oscillations of $CVA^t$ shown in Figure 8. Such oscillations are spurious and their presence is highly unattractive when CVA is computed and reported as part of daily profit and loss (P&L).

### 8.2 Brownian bridge method

Overcoming the deficiencies outlined in Section 8.1 through brute-force daily simulation is, unfortunately, prohibitively expensive for large portfolios, mostly due to
the expense of repricing the entire portfolio at each simulation path and each observation date. On the other hand, merely simulating risk factors at a daily resolution is generally feasible, as typically the number of simulated risk factors is relatively small (eg, several hundred) and the equations driving risk factor dynamics are usually simple. Further, having produced risk factor paths on a daily grid, we can normally also produce all realized trade flows along each path because trade flows, unlike trade prices, are usually simple functions of the realized risk factors.

Based on these observations, we propose the following algorithm for generating paths of portfolio values and trade flows on a daily time grid.

1. Simulate the paths of market risk factors with daily resolution.
2. For each path \( m \), use the simulated risk factors to calculate trade flows on the path with a daily resolution.
3. For each path \( m \) and each coarse portfolio valuation time point \( s_j \) (\( j = 1, \ldots, J \)), use the simulated risk factors to calculate portfolio value on the path \( V_m(s_j) \).
4. For each path \( m \) and each time point \( s_j \), use the trade flows realized on the path between times \( s_{j-1} \) and \( s_j \) to calculate the “forward” to the \( s_j \) portfolio values \( V_m'(s_{j-1}; s_j) \):

\[
V_m'(s_{j-1}; s_j) = V_m(s_{j-1}) - TF_{m}^{\text{net}}(s_j; (s_{j-1}, s_j)).
\]  

(8.1)
Note that $V'_m(s_{j-1}; s_j)$ is not a true forward value because we are subtracting the realized trade flows rather than forward trade flow values calculated at $s_{j-1}$.

(5) For each path $m$ and each exposure measurement time point $s_j$, compute the local variance $\sigma^2_m(t_{j-1})$ for the portfolio value “diffusion” $V_m(s_j) - V'_m(s_{j-1}; s_j)$ via a kernel regression estimator\(^{15}\) conditional on the realized value of $V'_m(s_{j-1}; s_j)$. Here, we use the term “diffusion” to indicate that the change in portfolio value has been defined to avoid any discontinuities resulting from trade flows.

(6) For each path $m$ and each exposure measurement time point $s_j$, simulate an independent, daily sampled, Brownian bridge process (see, for example, Glasserman 2004, Chapter 3) that starts with the value $V'_m(s_{j-1}; s_j)$ at time $s_{j-1}$ and ends at $V_m(s_j)$ at time $s_j$. The volatility of the underlying Brownian motion should be set equal to $\sigma_m(s_{j-1})$.

(7) For each path $m$ and each exposure measurement time point $s_j$, the portfolio values for each time $u$ of the daily grid in the interval $(s_{j-1}, s_j)$ are approximated from the simulated Brownian bridge $BB_m(u)$ by adding the trade flows realized along the path $m$ between times $u$ and $s_j$:

$$V_{m\text{approx}}(u) = BB_m(u) + TF_{m}^{\text{net}}(s_j; (u, s_j)).$$

(8.2)

In a nutshell, the algorithm above uses a Brownian bridge process to interpolate portfolio values from a coarse grid in a manner that ensures that intermediate trade flow events are handled accurately. The algorithm produces paths of portfolio values and trade flows on a daily time grid, wherefore exposure can be calculated as described in Section 7.3 with daily resolution and overlapping MPoRs. Further, daily sampling allows further refinements of the proposed model by consistently incorporating thresholds, a minimum transfer amount and rounding.

A key assumption made by the Brownian bridge algorithm is that the portfolio value process within the interpolation interval is a combination of an approximately normal “diffusion” overlaid by the realized trade flows. For Wiener process models without risk factor jumps, this assumption is accurate in the limit of infinitesimal interpolation interval, and it is often a satisfactory approximation for monthly or even quarterly interpolation steps. Nevertheless, the presence of trade flows that depend on values of risk factors between the end points introduces two types of error.

\(^{15}\) For example, the Nadaraya–Watson Gaussian kernel estimator (see Nadaraya 1964; Watson 1964). The selection of bandwidth for the kernels is covered in, eg, Jones\textit{et al} (1996). In our numerical results, we use Silverman’s “rule of thumb” (Silverman 1986).
(1) Suppose that there is a trade flow between the end points that depend on a risk factor value at the date when it is paid. The independence of the Brownian bridge process from the risk factor processes that drive that trade flow would result in an error in the expected exposure profile around the trade flow date. This error is the largest for trade flows in the middle of the interpolation interval and disappears for trade flows near the ends of the interval.

(2) Suppose that there is a trade flow that occurs after the end point of the interpolation interval, but whose value entirely depends on the value of a risk factor within the interpolation interval. A typical example would be a vanilla interest rate swap when the floating leg payment being paid after the end of the interpolation interval depends on the interest rate on a date within the interval. Even in the absence of a trade flow within the interpolation interval, the volatility of swap value drops at the floating rate fixing date as some of the uncertainty is resolved. Thus, the “true” swap value process has two volatility values: a higher value before the rate fixing date and a lower value after the rate fixing date. In contrast, the approximation algorithm assumes a single value of volatility obtained via kernel regression between the end points. Similarly to the decorrelation error discussed above, the error resulting from this volatility mismatch is the largest for fixing dates in the middle of the interpolation interval and disappears for fixing dates near the end points.

To illustrate the two errors above, Figure 9(a) shows the expected exposure profile for a one-year interest rate swap when a monthly grid for full valuation is situated so that the payment/fixing dates sit roughly in the middle of the interpolation interval, thus maximizing the error of the Brownian bridge algorithm. While there are, as expected, some errors around the trade flow dates, they are acceptable in magnitude and overall unbiased, in the sense that overestimation of exposure is about as frequent as underestimation of exposure. For, say, CVA purposes, the Brownian bridge results would therefore be quite accurate.

Figure 9(b) shows the expected exposure profiles when the monthly valuation points are aligned with rate fixing/payment dates. In this case, the Brownian bridge approximation is nearly exact. Of course, in practice such alignment is only possible for a single trade or a small netting set, and not for a large portfolio, where trade flows will occur daily. Yet, even for large netting sets the calculation accuracy will improve if interpolation pillars are aligned with the largest trade flows (e.g., the principal exchange date for the largest notional amounts). In practice, errors can typically be expected to be somewhere between the two extremes of parts (a) and (b) in Figure 9.

While the exact speedup provided by the Brownian bridge method depends on the implementation, for most portfolios the overhead of building the Brownian bridge at
a daily resolution is negligible compared with computing the exposure on the model’s coarse grid. In this case the computational effort of the daily Brownian bridge method is about half of the computational effort of the coarse grid lookback method, as the former does not require a “lookback” point to be added to each of the primary coarse grid points. Thus, the Brownian bridge method is both faster and significantly more accurate than the standard coarse grid lookback method.
The posting of initial margin (IM), in addition to regular variation margin (VM) collateral, provides banks with a mechanism to gain additional default protection. The practice of posting IM has been around for many years, typically with IM being computed at trade inception on a trade-level basis. This type of IM is entirely deterministic and normally either stays fixed over the life of the trade or amortizes down according to a prespecified schedule. As a consequence, modeling the impact on exposure is trivial: for the exposure points of interest, all trade-level IM amounts are summed across the netting set, and the total (which is the same for all paths) is subtracted from the portfolio value on each path.

A more interesting type of IM is dynamically refreshed to cover portfolio-level closeout risk at some high percentile, often 99%. This type of margin is routinely applied by central counterparty clearinghouses and margin lenders, and it will also soon be required by regulators for interdealer OTC transactions. In particular, in 2015 the Basel Committee on Banking Supervision (BCBS) and International Organization of Securities Commissions (IOSCO) issued a final framework on margin requirements (BCBS/IOSCO 2015), under which any two covered entities that are counterparties in noncentrally cleared derivatives are required to

(i) exchange VM under a zero-threshold margin agreement,

(ii) post IM to each other without netting the amounts.

IM must be held in a default-remote way, eg, by a custodian, so that IM posted by a counterparty should be immediately available to it should the other counterparty default.

Under the BCBS and IOSCO rules, regulatory IM can be calculated either by an internal model or by lookup in a standardized schedule. If an internal model is used, the calculation must be made at the netting set level as the value-at-risk (VaR) for a 99% confidence level. The horizon used in this calculation equals $9 + a$ business days, where $a$ is the remargining period (restricted to one business day under the US rules). Diversification across distinct asset classes is not recognized, and the IM internal model must be calibrated to a period of stress for each of the asset classes. The required levels of the IM will change as cashflows are paid, new trades are booked or markets move. To accommodate this, banks would either call for more IM or return the excess IM.

For trades made under BCBS and IOSCO rules, we must find a way to estimate the future IM requirements for each simulated path. No matter how simple the IM VaR

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16 Covered entities include all financial firms and systemically important nonfinancial firms. Central banks and sovereigns are not covered entities.
Rethinking the margin period of risk

model is, it will likely be difficult to perform such a calculation in practice if we want to incorporate with great precision all the restrictions and “twists” of the IM rules (stress calibration, limited diversification allowance, asset class mappings) as well as the fact that industry calculators (International Swaps and Derivatives Association 2016) tend to utilize standardized risk factors that may not correspond perfectly to those used in a given simulation framework.

The challenge of computing exposure and IM under the BCBS–IOSCO rules is discussed in much more detail in Andersen et al (2016), wherein a variety of numerical techniques are also provided. For our more modest purposes here, we will instead limit ourselves to an idealized IM calculation that will suffice to illustrate the main principles and complexities at play. To outline our assumptions, observe first that to calculate exposure at time \( t \), we have assumed that the last observation date for which \( C \) would deliver VM to \( B \) is \( t_C = t - \delta_C \). It is reasonable to assume that this date will also be the last observation date for which \( C \) would deliver IM to a custodian. To simplify modeling, we assume that the custodian would not return any amount of IM to \( C \) for observation dates after \( t - \delta_C \). Thus, to calculate exposure at time \( t \), IM on a path has to be estimated from the dynamics of the exposure model as of time \( t - \delta_C \).

Assuming, as is common in practice, that portfolio values are locally Gaussian, it suffices to know the local volatility of the portfolio value for the period \([ t - \delta_C, t] \) estimated at \( t - \delta_C \). Denoting the IM horizon by \( \delta_{IM} \) and the local volatility of portfolio value at time \( u \) on path \( m \) via \( \sigma_m(u) \), the IM available to \( B \) at the ETD date \( t \) on path \( m \) is given by

\[
IM_m(t - \delta) = \sigma_m(t - \delta) \sqrt{\delta_{IM}} \Phi^{-1}(q),
\]

where \( q \) is a confidence level (often 99%) and \( \Phi^{-1}(\cdot) \) is the inverse of the standard normal cumulative distribution function.

Estimation of the local volatility \( \sigma_m(t - \delta) \) can be done via kernel regression, as in Section 8.2. If the portfolio value is simulated at both \( t - \delta_C \) and \( t \), the kernel regression for \( \sigma_m(t - \delta) \) could be run on the P&L \( V(t) - V(t - \delta_C) + TF_{net}(t; t - \delta_C, t) \) conditional on realization of the portfolio value on path \( m \) at the beginning of the MPoR, \( V_m(t - \delta_C) \). If we do not calculate the portfolio value at the beginning of the MPoR, but use the fast approximation of Section 8.2 instead, \( \sigma_m(t - \delta) \) can be set equal to the local volatility estimated for the time interval that encloses the given MPoR \([ t - \delta_C, t] \). Thus, our Brownian bridge framework can now produce not only the collateralized exposure under VM alone, but also a reasonable estimate of the collateralized exposure under a combination of VM and IM.

In calculating the impact of IM, an important consideration is the timing and mechanics of adjustment to the IM when \( C \) misses a margin flow or a trade flow. For instance, when a large trade reaches maturity, the portfolio VaR may be reduced, in which case some of the IM posted by \( C \) must be refunded. The issue of whether this
As evident from the figure, the IM mechanism strongly reduces exposure away from trade flows, but near trade flow dates the protection gets progressively weaker toward trade maturity, and disappears almost completely for the last couple of trade flows. The reason for this uneven benefit of IM on this trade is that the ten-day VaR of the trade bears no direct relationship to the size of trade flows that determine exposure spikes in our model. The variance of the P&L is reduced as the swap approaches maturity, so the amount of IM on a given path is also reduced. However, the size of the trade flows is not reduced, but can actually grow with the simulation time (as larger and larger realizations of the floating rate are possible). Thus, when the swap approaches maturity, the amount of IM is greatly reduced relative to the trade flows, so exposure spikes grow larger, while the “diffusion” component of exposure becomes smaller.

The impact of IM on the vanilla swap and cross-currency swap portfolios described in Section 7.3 is shown in Figure 11. As in Figure 10, the IM strongly suppresses the “diffusion” component of portfolio value changes but proves inadequate in reducing refund can be delayed due to an ongoing margin dispute is not yet fully resolved. To simplify the calculations, we have assumed that no part of IM is returned to C during the MPoR.

To show some numerical results, we can consider the individual trades and portfolios in Section 7.3. Our first example uses the ten-year vanilla swap from Figure 4, for which the impact of IM on exposure is shown in Figure 10.

As evident from the figure, the IM mechanism strongly reduces exposure away from trade flows, but near trade flow dates the protection gets progressively weaker toward trade maturity, and disappears almost completely for the last couple of trade flows. The reason for this uneven benefit of IM on this trade is that the ten-day VaR of the trade bears no direct relationship to the size of trade flows that determine exposure spikes in our model. The variance of the P&L is reduced as the swap approaches maturity, so the amount of IM on a given path is also reduced. However, the size of the trade flows is not reduced, but can actually grow with the simulation time (as larger and larger realizations of the floating rate are possible). Thus, when the swap approaches maturity, the amount of IM is greatly reduced relative to the trade flows, so exposure spikes grow larger, while the “diffusion” component of exposure becomes smaller.

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FIGURE 10 Expected exposure for ten-year vanilla swap.

US$10 million ten-year payer swap, 2% coupon; fixed coupons paid semiannually; floating coupons paid quarterly. Model and exposure setup as in Figure 4, except that the graph “swap with IM” uses risk-based IM to protect the swap, per (9.1) with $q = 99\%$. The Conservative exposure model calibration is used everywhere.
FIGURE 11  Expected exposure for swap and XCCY swap portfolios.

(a) Regular interest rate swaps. (b) Cross-currency swaps. Exposure profiles are shown for the portfolios in Figures 6 and 7, respectively. Each part contains exposures both with and without IM protection.

the spikes of exposure for both the single currency and, in particular, the cross-currency portfolio.

10 CONCLUSION

Industry standard models for collateralized credit risk are well known to produce nonnegligible counterparty credit exposure, even under full variation margin collateralization. This exposure essentially arises due to inevitable operational and legal
delays (the margin period of risk) that are “baked into” the workings of the ISDA contracts that govern OTC trading.

In the most common industry implementation, the length of the MPoR, and what precisely transpires inside it, is, however, often treated in a highly stylized fashion. Often the MPoR is set equal to ten business days for no reason other than tradition, and often counterparties are assumed to have oddly synchronized behavior inside the MPoR. For instance, one common approach (denoted Classical−) assumes that margin and trade flows by both counterparties terminate simultaneously at the beginning of the MPoR. Another approach (Classical+) assumes that margin flows terminate at the beginning of the MPoR, but trade flows terminate (simultaneously) at the end of the MPoR. Surprisingly, the Classical+ and Classical− approaches continue to coexist in the market, and neither have become the sole market practice. We speculate that one reason for this state of affairs is that the two models correspond to different choices for a trade-off between implementation complexity and model stability: Classical+ is easier to implement but it is prone to spurious spikes in daily CVA P&L (as demonstrated in Section 8.1), whereas the Classical− model is more difficult to implement but is free from such spikes.

Ultimately, a model should, of course, be selected not on the basis of ease of implementation or the properties of a specific numerical technique, but on how well the model captures the legal and behavioral aspects of events around a counterparty default. To this end, even a cursory analysis suggests that the perfect synchronicity of the Classical± methods cannot be supported in reality. For instance, due to the ways in which a CSA works in practice, a nondefaulting party will need at least three business days after a portfolio valuation date to determine for sure that the corresponding margin payment by its counterparty will not be honored.

In this paper, we carefully dissected the MPoR into a full timeline around the default event, starting with a missed margin call and culminating on the post-default valuation date on which the termination value of the portfolio is established. For modeling purposes, we condensed this timeline into four model parameters, each specified as the number of business days prior to termination for the following events:

1. the last market data measurement for which the margin flow is received ($\delta_C$) and paid ($\delta_B$) as prescribed;
2. the last date when the defaulting party ($\delta'_C$) and the bank ($\delta'_B$) make trade flow payments as prescribed.

17 The term “well” can mean different things in different applications of the exposure model. For regulatory capital purposes, prudence and conservatism may, for instance, be as important as outright precision.
18 In contrast, the classical model has only one parameter: the full length of the MPoR.
As we show, each of these four parameters has a precise legal and/or operational interpretation, enabling calibration from the terms of the CSA and from the operational setup of the bank. Note that the proposed model parameterization includes Classical+ and Classical− models as limit cases.

For indicative purposes, we describe two particular calibration choices for our model, denoted Aggressive and Conservative. The former assumes that the non-defaulting bank always operates at an optimal operational level and will enforce the legal provisions of the ISDA legal contracts as strictly as possible. The latter allows some slack in the operations of the bank, for manual checks of calculations, legal reviews, “gaming” behavior of the counterparty and so forth. The Conservative model setting obviously produces higher exposure than the Aggressive setting; this is for the following reasons:

1. the Conservative setting has a longer overall length of MPoR;
2. the Conservative setting has a longer margin flow gap period where the bank pays, but does not receive, margin flows;
3. the Conservative setting, unlike the Aggressive setting, contains a trade flow gap period where the bank pays but does not receive trade flows.

In our numerical tests, the first two factors cause the Conservative setting to have approximately twice the exposure of both the Aggressive setting and the Classical± settings away from the dates of large trade flows. The last factor (ie, the presence of a trade flow gap) may cause exposure spikes of extremely large magnitude under the Conservative calibration. Despite the fairly short duration of these spikes, they may easily add up to very significant CVA contributions, especially for cross-currency trades with principal exchange (“Herstatt risk”). Credit losses due to such trade flow gaps materialized in practice during the financial crisis (especially due to the Lehman Brothers default), so their incorporation into our exposure model is both prudent and realistic.

Detailed tracking of margin and trade flow payments requires stochastic modeling of the trade portfolio on a daily grid. As brute-force simulations at such a resolution are often impractically slow, it is important that numerical techniques be devised to speed up calculations. While the focus of our paper is mainly on establishing fundamental principles for margin exposure, we also proposed an acceleration method based on kernel regression and a Brownian bridge applied to portfolio values “stripped” of cashflows. For ordinary and cross-currency swaps, we demonstrated that this method is both accurate and much faster than either brute-force simulation or standard acceleration techniques for the classical model. Further improvements in acceleration techniques, and expansion of applicability into more exotic products, is an area of future research.
Under suitable assumptions, kernel regression may also be used to embed risk-based initial margin into the exposure simulation. As we demonstrated in Section 9, initial margin at a 99% level succeeds in greatly reducing the bilateral exposure for the Classical—model calibration. For all other calibration choices, and especially for the Conservative setting, the reduction in counterparty exposure afforded by the initial margin fails around the time of large trade flows, when the sudden change in exposure following a trade flow exceeds the initial margin level. Note that the (already inadequate) level of IM protection around trade flows deteriorates further toward the maturity of the portfolio, where the local volatility of the portfolio value decreases but trade flows do not. Overall, accurate modeling of events within the MPoR becomes critically important for portfolios covered by dynamic IM.

Finally, let us emphasize that our definition of credit exposure in this paper is tied to the legal workings of the IMA/CSA framework, with the MPoR essentially being the time needed to legally terminate trades held with a nonperforming counterparty. In contrast, the regulatory definition of MPoR (Basel Committee on Banking Supervision 2010) also includes the time taken to fully replace (or re-hedge) a defaulted portfolio. For illiquid or hard-to-trade securities, this may add a nontrivial amount of time to the MPoR. In practice, of course, losses in a netting set will often span many independently operated trading books, the traders of which would never replace or re-hedge a defaulted portfolio in a simplistic trade-by-trade fashion. Still, to the extent that some element of re-hedging costs can be successfully defended as being part of the bankruptcy costs, we could imagine extending our framework to cover liquidity risks by either extending the MPoR (to cover the situation where obtaining the dealer-poll for a portfolio takes longer than normal) or impairing the portfolio value (to cover the situation where liquidity charges will adversely affect the portfolio value). We leave such extensions to future research.

**DECLARATION OF INTEREST**

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

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Research Paper

Creditwatches and their impact on financial markets

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ABSTRACT

Credit rating agencies (CRAs) monitor a firm’s creditworthiness and place firms on creditwatch when they observe potential changes in firm characteristics. However, prior research largely focuses on a creditwatch as a single event and neglects the fact that a rating continues to be “on watch” until a final decision is made. This paper examines the impact of creditwatch placements on a firm’s credit default swap (CDS) spread and its stock price during the time interval between the creditwatch placement and the final rating decision. The investigation includes 311 rating creditwatch placements (204 negative and 107 positive) from July 2006 to June 2014. The results indicate that stock returns are not different from zero for negative and positive creditwatch placements, whereas CDS spreads continuously increase between a negative creditwatch placement and the final decision. This is a novel result, and therefore creditwatch placements may potentially explain why prior studies found that CDS markets anticipate rating changes. However, this increase depends on the reason for the creditwatch announced by the CRA. The market reaction is strongest for firm-specific performance announcements that may affect the future cashflow development of a company.

Keywords: creditwatch; credit ratings; credit risk; credit default swaps; stock performance.
1 INTRODUCTION

For several decades now, credit rating agencies (CRAs) have been considered important actors in financial markets. Credit ratings are important for investment decisions made by institutional investors and allow a firm’s credit risk to be understood via a simple rating code. Research shows that investors rely on the CRAs’ announcements and their evaluation procedures for investment decisions (Bannier and Hirsch 2010; Hand et al. 1992; Holthausen and Leftwich 1986; Lehnert and Neske 2006). However, CRAs not only disclose information to investors but have a monitoring role as well, which is most apparent in their “creditwatch” procedure (Boot et al. 2006).1 Creditwatch procedures give indications of possible credit rating changes in the future. When a CRA finds new information that could potentially change the firm’s perceived risk characteristic, it notifies the firm’s management and asks for clarification. During the creditwatch period, the firm’s future risk profile and the final rating decision are uncertain. The firm’s credit risk uncertainty, and therefore the creditwatch score, can have several causes, e.g., the announcement of a new corporate strategy, trends in the firm’s financial conditions or changes in the operating performance (Goh and Ederington 1993, 1999; Imbierowicz and Wahrenburg 2013). In addition, the recent financial crisis, with its peak in 2008, indicates that several external creditwatch and rating causes exist (e.g., sovereign downgrades2).

The literature shows that creditwatch announcements have a significant effect on stock prices (Bannier and Hirsch 2010; Boot et al. 2006; Galil and Soffer 2011; Goh and Ederington 1993, 1999; Gropp and Richards 2001; Hand et al. 1992). The creditwatch announcements of CRAs even induce stronger market reactions than the subsequent rating changes (Kiesel and Schiereck 2015; Norden and Weber 2004). Bannier and Hirsch (2010) show that the creditwatch allows CRAs to enter a contract with their rated firms. This finding is based on a theoretical model by Boot et al. (2006), who conclude that a CRA acts as a coordinator between firms and their investors.

Empirical research largely analyzes creditwatch placements as single events and neglects the fact that a rating continues to be “on watch” until a final decision is made. This paper contributes to the literature in two different ways. First, this study focuses on the changes in credit default swap (CDS) spreads and stock prices between the creditwatch placement and the final rating decision. The creditwatch placement possesses risk-relevant information for market participants, who are faced with uncertainty as to whether the firm’s actions to prevent the potential downgrade will be

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1 Standard & Poor’s (S&P) puts firms on “Creditwatch”, Moody’s uses a “Watchlist” and Fitch has “Rating Watches” to indicate that there is a heightened probability of a rating change. In this paper, we use the terminology “watchlist”, “review” and “creditwatch” interchangeably.

2 Almeida et al. (2016) find that sovereign downgrades cause the downgrade of a firm whose rating was equal to or above the sovereign one prior to the downgrade.
successful. Prior to the resolution of this uncertainty by the final decision of the CRA, investors have to evaluate the new information. We examine the stock market in conjunction with the CDS market in order to assess the impact of creditwatch placements on equity and credit markets. Similarly to CRAs, the CDS market contains valuable information about the credit risk of the bond issuer. Hull et al (2004) and Norden and Weber (2004) find that CDS markets are able to anticipate the CRA action, as CDS spreads generally increase before the CRA announces a rating action. This effect has been confirmed by more recent studies (see, for example, Finnerty et al 2013; Galil and Soffer 2011).

Second, this study investigates whether creditwatch placements due to firm-specific causes, such as changes in firm performance or capital structure, are more difficult to anticipate for investors than placements due to changes in the market environment, which affect all firms in a similar fashion. By analyzing the CDS spread change and stock performance for a sample of 311 S&P creditwatch placements and analyzing the cause of the creditwatch placement, we are able to test whether the review process serves different purposes, depending on the reason for the creditwatch.

The results indicate that creditwatch procedures do not have an economic impact on equity markets. The CDS spreads, as a measure of risk, show that the firm’s risk significantly increases until the final rating downgrade. The “anticipation effect” suggested by Hull et al (2004) may be explained by the creditwatch because participants in the CDS market adjust their assessment of a firm’s risk while the rating is on creditwatch. In addition, the results show that the CDS spread increase depends on the reason for the creditwatch released by the CRA. The CDS market reacts more strongly to firm-specific performance announcements, which may affect the future cashflow development of a company.

The rest of this paper is structured as follows: Section 2 reviews the related literature, focusing on the influence of creditwatch placements and rating announcements as single events. Section 3 describes the methodology, data and sample selection process. Section 4 presents the findings of the empirical investigation. Section 5 offers multiple robustness checks and Section 6 concludes the paper.

2 LITERATURE REVIEW

Credit ratings have a long research history. The literature can be divided into three major fields of research:

(i) the impact of creditwatches on investors;

(ii) the relationship between CDS spreads and credit rating announcements;

(iii) the impact of creditwatch placements and rating changes on stock returns.
Boot et al (2006) analyze the monitoring role of CRAs and their creditwatch procedure as a coordination mechanism for investor beliefs. They show in their theoretical model that CRAs add a monitoring-type element to the financial markets. CRAs play an important role as a “focal point” in resolving failures in coordination between multiple investors. The ability to resolve such coordination failures arises from the effects of the CRAs’ actions, rating changes and creditwatch placements. CRAs help investors by coordinating their investment decisions. The creditwatch mechanism allows CRAs to influence the firm’s risk choices by threatening them with rating downgrades and subsequent investor reactions. Boot et al (2006) show that there exists an “implicit contract” between CRAs and the firms they rate.

Bannier and Hirsch (2010) analyze the economic impact of creditwatch announcements by Moody’s. They compare the effect of rating changes by Moody’s before and after the introduction of its institutionalized creditwatch process (the “Watchlist”) in 1991. The information contained in rating changes following the introduction of the creditwatch process leads to stronger market reactions than in the years prior to the introduction of the creditwatch. They confirm the results of the model by Boot et al (2006): the creditwatch process leads to an implicit contract between the CRA and the rated firms, particularly for non-investment-grade firms. Chan et al (2011) also analyze the model of Boot et al (2006), using Moody’s ratings from 1990 to 2006. They are not able to confirm that rating downgrades preceded by a placement on creditwatch are more informative than those downgrades not preceded by a creditwatch process. Rating affirmations of firms whose rating is on creditwatch do not lead to statistically significant equity market reactions.

Academic research on CDSs is comparatively new. CDSs were created in the early 1990s, but the CDS market did not start to play a bigger role in financial markets until 2000. The CDS market is an over-the-counter (OTC) market, and the availability of CDSs increased after 2000. The relationship between CDSs and stock markets is one area of recent research (Acharya and Johnson 2007; Blanco et al 2005; Hull et al 2004; Kiesel et al 2016; Longstaff et al 2005; Norden and Weber 2009). Acharya and Johnson (2007), as well as Norden and Weber (2004) and Hull et al (2004), find that CDS markets appear to anticipate rating changes prior to stock markets. Therefore, there appears to be an information flow from the CDS to the stock market.

Hull et al (2004) examine the relationship between credit ratings and CDS spreads and find that CDS spreads predict negative rating events. They show that, prior to negative outlooks, negative creditwatch placements and rating downgrades, CDS spreads increase significantly. Yet they find no such effects for positive outlooks, positive creditwatch placements or rating upgrades. They conclude that an anticipation effect does exist in the CDS market, albeit only for negative rating events. Moreover, they document that an announcement effect during the $[-1; +1]$ day event window only
exists for negative creditwatch announcements. Therefore, Hull et al argue that creditwatch announcements in particular contain significant information for credit market participants, whereas downgrades and negative outlooks do not.

Norden and Weber (2004) use a similar methodology for the period between July 1998 and December 2002. They find a significant negative effect for downgrades on the CDS and stock markets for S&P, Moody’s and Fitch rating announcements. They also show an anticipation effect for CDS and stock markets. Negative creditwatch placements have a larger economic impact than downgrades themselves.

Finnerty et al (2013) conduct an analysis of S&P rating actions between 2001 and 2009. In contrast to prior studies, they find significant CDS market reactions to rating upgrades. Yet, overall, they confirm that downgrades have a greater impact on CDS spreads than upgrades. Galil and Soffer (2011) research the CDS market in greater detail with regard to rating announcements. They show that bad news and negative rating announcements tend to cluster, whereas positive rating announcements are less frequent. They confirm the previous findings that the CDS market responds more strongly to bad news. Due to the clustering of bad news, they argue that the rating changes themselves contain only comparatively little information.

Imbierowicz and Wahrenburg (2013) investigate whether there are wealth transfers between stock holders and bondholders following Moody’s rating downgrades and creditwatch placements. Their results show the negative effect of downgrades and negative creditwatch announcements on CDS and stock markets and they find a wealth transfer from equity holders to bondholders, which is particularly pronounced if mergers and acquisitions are the reason for the downgrade or the creditwatch placement. In addition, they analyze the rationale for creditwatch placements and rating changes by grouping the creditwatch announcements and the rating changes into different categories. Downgrades for all categories have a significant effect on the CDS market, except for those attributable to changes in the capital structure. Imbierowicz and Wahrenburg (2013) conclude that the reason for the rating is an important factor in market participants’ evaluation of the impact of rating changes on a firm’s credit risk.

The CDS literature has so far treated creditwatch placements and rating changes as two separate events. However, ratings continue to be “on watch” for a certain period. CRAs place the rating on a formal creditwatch and communicate this to the market when they observe deviations due to changes in firm characteristics or certain events that may potentially change its creditworthiness, and thus its credit rating in the near future. The time period for which a rating is placed on the creditwatch is characterized by uncertainty regarding the firm’s creditworthiness. If there is no uncertainty, the new information on the firm should be immediately correctly evaluated by the market participants. Wansley and Clauretie (1985) and Bannier and Hirsch (2010) show for the equity market that creditwatch placements are connected to the actual rating changes. Therefore, we extend the previous CDS literature by analyzing the
entire duration of a firm’s rating on the CRA’s creditwatch. Hull et al (2004) and Norden and Weber (2004) find anticipation effects for the CDS market with regard to negative creditwatch placements. If there are anticipation effects in the CDS market, uncertainty with regard to the ultimate outcome of the rating review should not be observed. As a result, we propose the following hypothesis.

(H1) Firms placed on negative (positive) creditwatch with a subsequent downgrade (upgrade) experience a significant increase (decrease) in their CDS spread level during the time interval between the creditwatch placement and the final rating decision.

Next, we analyze the firm’s share price during the time interval between the creditwatch placement and the final rating decision. Negative market reactions on stock markets after rating downgrades are well documented (see, for example, Bannier and Hirsch 2010; Goh and Ederington 1993, 1999; Gropp and Richards 2001; Hand et al 1992; Kiesel and Schiereck 2015; Lehnert and Neske 2006). In contrast to downgrades, the findings on rating upgrades are inconclusive. Holthausen and Leftwich (1986), Goh and Ederington (1993, 1999) and Bannier and Hirsch (2010) find insignificant market reactions to rating upgrades; Jorion and Zhang (2007) and Dichev and Piotroski (2001) are able to empirically verify a weak reaction on the stock and bond markets. Jorion et al (2005) find significant positive market reactions following rating upgrades after SEC Regulation Fair Disclosure became effective in October 2000. In addition to rating downgrades, rating reviews for downgrades also lead to significant negative stock market reactions (Hand et al 1992; Holthausen and Leftwich 1986). Holthausen and Leftwich (1986) document significant positive equity market reactions to rating reviews for upgrades, while Hand et al (1992) find the opposite to be true, as they find significant negative stock market reactions for rating reviews for downgrades. The literature indicates that the stock market and the CDS market react similarly to rating announcements. This leads to the following hypothesis.

(H2) Firms placed on negative (positive) creditwatch with a subsequent downgrade (upgrade) experience a significant decrease (increase) in their stock price during the time interval between the creditwatch placement and the final rating decision.

In line with Imbierowicz and Wahrenburg (2013), we also investigate the reason for creditwatch placements. Imbierowicz and Wahrenburg show that the CDS market responds differently depending on the reason for the announcement. Creditwatch placements attributable to changes in capital structure do not lead to significant market reactions, while other causes have a significant influence on CDS spreads. Goh and Ederington (1993, 1999) show similar results for the stock market. They find
that rating downgrades as a result of a deterioration in a firm’s earnings or financial prospects lead to significant stock market reactions, while downgrades owing to changes in a firm’s leverage or for other reasons do not. Goh and Ederington (1993) consequently conclude that only rating announcements regarding a firm’s financial prospects provide new information to market participants; other rating changes do not. As a result, we expect that the probability of a rating affirmation or rating change will at least partly depend on the creditwatch rationale. Such an effect may also explain the anticipation effects observed in prior studies focusing on CDS markets. This leads to the following hypothesis.

(H3) CDS spread increases (decreases) and abnormal stock returns for firms placed on creditwatch negative (positive) depend on the rationale for the creditwatch.

3 DATA AND METHODOLOGY

3.1 Data sample and selection

The analysis is based on an international sample of European and US listed firms with available CDS spread data and a long-term issuer rating by S&P.3 CDS data is based on the Thomson Reuters Composite CDS data and combines Thomson Reuters and credit market analysis (CMA) data using the splice function. The data covers the period from July 1, 2006 to June 30, 2014. Single-name CDSs have generally become more liquid instruments, and we forgo the common practice of linearly interpolating daily mid CDS spreads between missing spread change observations (Finnerty et al 2013; Hull et al 2004; Norden and Weber 2004). Further, we exclude banks, financial services and insurance companies (Standard Industrial Classification 6000–6999) for this investigation, because they have a unique capital structure, which affects their performance differently. After excluding sovereigns and finance-related companies, we match CDS data to its respective companies. In total, 497 different firms have a long-term issuer rating by S&P, CDS and stock data. The S&P press releases are collected from Alacra. We are interested in the time between a creditwatch announcement and its subsequent rating decision. We control for contaminated rating announcements and exclude all creditwatch placements combined with a rating change. The rating change has to be a subsequent downgrade for firms placed on creditwatch negative or a subsequent upgrade for firms on creditwatch positive. In total, 218 of the 497 firms with a long-term issuer rating by S&P were placed on creditwatch: 154 firms from the US and 64 from Europe. S&P says its rating decision announcement is usually ninety days after the creditwatch placement, but their decision depends on the reason

3 S&P is arguably the biggest and most influential CRA and has a market share in corporate issuer ratings of approximately 46% in the US and 35% in Europe. The study by Finnerty et al (2013) also exclusively focuses on the impact of S&P rating announcements on CDS spreads.
for the creditwatch placement and the amount of time taken to obtain and analyze the information. The median number of trading days between a creditwatch placement and the final rating decision is fifty-one in the sample; the mean is sixty-three trading days. The final rating decision is faster for firms placed on creditwatch positive (fifty-five trading days) than for those placed on creditwatch negative (sixty-eight trading days). This suggests that positive information is easier to evaluate. Table 1 provides the summary statistics of the final data set. We found 204 negative creditwatch placements with a subsequent downgrade and 107 positive creditwatch placements with a subsequent upgrade.

There can be several reasons for the firm’s credit risk uncertainty and therefore its creditwatch placement, e.g., the announcement of a new firm strategy, trends in the firm’s financial conditions or changes in operating performance. In addition, the 2007–9 financial crisis showed that creditwatch placement and rating changes can have external causes (e.g., sovereign downgrades or weak market demand). Following Goh and Ederington (1993, 1999) and Imbierowicz and Wahrenburg (2013), we analyze the reasons given in the CRA’s press releases; reasons for being placed on creditwatch are classified according to keywords in the S&P press release. In line with Goh and Ederington (1993), performance causes indicate possible changes in the future cashflow development (e.g., firm strategy, future sales), whereas capital structure causes are one-off changes in the firm’s leverage (e.g., capital increases, bond issues). The third creditwatch cause is related to the market environment (e.g., market turmoil) and is particularly relevant for negative creditwatch placements.

Table 1 also displays the number of rating events by cause for the creditwatch placement. Most creditwatch placements with a subsequent rating change are attributed to changes in the firm’s performance and changes in capital structure. This is in line with the previous findings (Goh and Ederington 1993, 1999; Imbierowicz and Wahrenburg 2013). Due to the macroeconomic environment, we find fifty-seven negative creditwatch placements with a subsequent downgrade. Other reasons for creditwatch placement, such as changes in the rating methodology of S&P, occur only for positive creditwatch placements.

The distribution of negative and positive creditwatch placements by year and rating class is summarized in Table 2. Most of the negative creditwatch placements can be observed during the peak of the financial crisis. In the aftermath of the crisis, negative creditwatch placements are rare. Most positive creditwatch placements can be observed one year prior to the crisis and in the year after the financial crisis.

### 3.2 Methodology

The methodology to measure the abnormal CDS spread changes is analogous to that in prior studies (see, for example, Finnerty et al 2013; Hull et al 2004; Norden and...
### TABLE 1  Descriptive analysis of the sample.

#### (a) Sample statistics

<table>
<thead>
<tr>
<th>Creditwatch placements</th>
<th>Firms</th>
<th>Time on watchlist (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US</td>
<td>Europe</td>
</tr>
<tr>
<td>Total</td>
<td>223</td>
<td>88</td>
</tr>
<tr>
<td>Downgrades</td>
<td>138</td>
<td>66</td>
</tr>
<tr>
<td>Upgrades</td>
<td>85</td>
<td>22</td>
</tr>
</tbody>
</table>

#### (b) Reason for creditwatch placement

<table>
<thead>
<tr>
<th>Performance</th>
<th>Capital structure</th>
<th>Market</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>175</td>
<td>66</td>
<td>61</td>
<td>9</td>
</tr>
<tr>
<td>Downgrades</td>
<td>107</td>
<td>40</td>
<td>57</td>
<td>0</td>
</tr>
<tr>
<td>Upgrades</td>
<td>68</td>
<td>26</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

This table shows the descriptive sample statistic of the sample of 311 creditwatch announcements. The events are divided into US and European firms, and the mean and median times of the creditwatch period are provided. In addition, the creditwatch placements are divided according to performance, capital structure, market conditions and other factors.

### TABLE 2  Number of creditwatch events by year of creditwatch placement.

<table>
<thead>
<tr>
<th></th>
<th>Creditwatch negative</th>
<th>Creditwatch positive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n  AAA–AA</td>
<td>A</td>
</tr>
<tr>
<td>2006</td>
<td>8  0</td>
<td>5</td>
</tr>
<tr>
<td>2007</td>
<td>30 2</td>
<td>11</td>
</tr>
<tr>
<td>2008</td>
<td>50 1</td>
<td>12</td>
</tr>
<tr>
<td>2009</td>
<td>42 1</td>
<td>7</td>
</tr>
<tr>
<td>2010</td>
<td>17 0</td>
<td>8</td>
</tr>
<tr>
<td>2011</td>
<td>25 1</td>
<td>11</td>
</tr>
<tr>
<td>2012</td>
<td>17 1</td>
<td>3</td>
</tr>
<tr>
<td>2013</td>
<td>12 0</td>
<td>4</td>
</tr>
<tr>
<td>2014</td>
<td>3 0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>204 6</td>
<td>61</td>
</tr>
</tbody>
</table>

This table shows the distribution of the 311 creditwatch placements in the sample. The table is split according to the initial rating by S&P. The leftmost column indicates the year of the creditwatch placement.
Weber 2004). CDS spread changes are adjusted by changes in a CDS spread index of the same rating class as the company’s initial rating:

$$\text{ASC}_{i,t} = (\text{CDS}_{i,t} - \text{CDS}_{i,t-1}) - (I_t - I_{t-1}),$$  \hspace{1cm} (3.1)

where $\text{ASC}_{i,t}$ is the abnormal CDS spread change for firm $i$ on day $t$, $\text{CDS}_{i,t}$ is the observed CDS spread for firm $i$ on day $t$, and $I_t$ is the CDS spread index on day $t$. Daily CDS spread index levels correspond to the equally weighted cross-sectional mean of all CDS spreads for each of the six rating classes (AAA/AA, A, BBB, BB, B, C).\(^4\) In order to test whether the adjusted CDS spread changes differ significantly from zero, we use the standard cross-sectional parametric $t$-test as well as the nonparametric Wilcoxon signed-rank test.

The relationship between CRA announcements and stock returns is analyzed using the market model event study introduced by Dodd and Warner (1983) and Brown and Warner (1985). The cumulative abnormal returns (CAR) of firm $i$ in the creditwatch period are calculated as

$$\text{CAR}_{j,[\tau_1, \tau_2]} = \sum_{t=\tau_1}^{t+\tau_2} R_{j,t} - E(R_{j,t}),$$  \hspace{1cm} (3.2)

where $R_{j,t}$ is the stock return of firm $j$ on day $t$ and $E(R_{j,t})$ is the daily expected return of stock $j$ calculated by the market model estimated over a one-year period (252 trading days) that ends three trading days before the announcement of the CRA. Raw stock returns are adjusted using the Datastream national total return index for the specific country.

In order to test whether the abnormal returns are statistically different from zero, we use, in analogy to the CDS spreads, two different test procedures. The Boehmer–Musumeci–Poulsen (BMP) test is a parametric test that restandardizes the CAR with cross-sectional standard deviation (Boehmer et al 1991). The Corrado–Zivney (CZ) test statistic is a nonparametric rank test (Corrado and Zivney 1992); it applies restandardized event windows and is robust against event-induced volatility and cross-correlation.

The time taken from the rating review announcement to the final rating decision varies across the sample and may depend on the cause of the review placement and the amount of time the CRA needs to obtain and analyze the relevant information. S&P states that its rating decision is usually reached within ninety days of placing a rating under formal review. Consequently, the standard procedure for analyzing calendar time returns during the time that a rating is on creditwatch is inappropriate. We apply the approach of Malmendier et al (2016) and normalize the creditwatch

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\(^4\) Due to the small sample sizes of the AAA and AA rated companies, these two classes are combined.
period to a relative time $t_R$, ie, between $t_R = 0\%$ and $t_R = 100\%$. The synthetic creditwatch period is set to the median number of trading days in the sample.\(^5\) For each creditwatch placement, we either condense the abnormal CDS spread change and the abnormal stock return if the time interval is longer than the synthetic period of fifty-one trading days, or we extend them to fifty-one trading days if the empirical event window has less than the median number of trading days. Day 51 of the synthetic creditwatch period is indicated by 100\% (relative time $t_R = 100\%$), whereas the day of the creditwatch announcement is indicated by 0\% (relative time $t_R = 0\%$).

We linearly interpolate the cumulative abnormal CDS spreads and the cumulative abnormal returns for the event-specific event window, denoted by $T_i$, beginning on the day of the creditwatch announcement and ending on the day of the rating change announcement. For example, if the CRA needs twenty days, ie, $T_i = 20$, between the creditwatch announcement and the final rating announcement in order to evaluate the event information, the synthetic cumulative abnormal CDS spread change after $t_R = 10\%$ relative time, $\overline{\text{CASC}}_i(10\%)$, is equal to the cumulative abnormal CDS spread change after $20 \times 10\% = 2$ trading days, ie, $\text{CASC}_i(t_R T_i)$. If the creditwatch duration is not an integer, $\overline{\text{CASC}}_i$ is calculated using the linear interpolation between the actual trading days (Malmendier et al 2016):

$$\overline{\text{CASC}}_i(t_R) = (1 - w_{(i,t_R)})\text{CASC}_i([t_R T_i]) + w_{(i,t_R)}\text{CASC}_i([t_R T_i] + 1), \quad (3.3)$$

where $\overline{\text{CASC}}_i$ is the synthetic cumulative abnormal CDS spread of firm $i$, $\lfloor \cdot \rfloor$ refers to the floor function ($w_{(i,t_R)} = t_R T_i - [t_R T_i]$) and $t_R$ and $T_i$ are as defined above. The linear interpolation of noninteger cumulative abnormal returns is analogous. We use the standard cross-sectional parametric $t$-test, as well as the nonparametric Wilcoxon signed-rank, to test whether the standardized CDS spreads ($\overline{\text{CASC}}$) and standardized returns ($\overline{\text{CAR}}$) differ significantly from zero.

4 EMPIRICAL RESULTS

Figure 1 gives a detailed view of the CDS and stock markets’ performance around creditwatch announcements (CW) and final rating decisions (D). The figure presents the $[-2; +30]$ event window around creditwatch placements and the $[-30; +2]$ event window around the final rating decision. The results are split into negative and positive creditwatch placements. For the CDS market, a negative creditwatch placement results in a sudden increase in CDS spreads. Positive creditwatch announcements lead to a CDS spread decrease in the short term, but in the long term the CDS spreads increase again. The figure also illustrates that the CDS spreads increase around the time of the final rating decision. Figure 1(b) illustrates the results for the stock market around

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\(^5\) Results are robust using the mean creditwatch period.
FIGURE 1  CDS and stock performance around creditwatch placement and final rating decision.

Results of (a) the CDS spread and (b) the stock performance for the entire sample of 311 creditwatch placements around the $[-2; +30]$ interval of the creditwatch placements (CW) and the $[-30; +2]$ interval around the creditwatch outcome, divided into reviews for downgrade and upgrade.

FIGURE 2  CDS and stock performance during the time on creditwatch.

Results of (a) the CDS spread and (b) the stock performance for the entire sample of 311 creditwatch announcements throughout the time that a rating is on review, divided into reviews for downgrade and upgrade.

creditwatches and final rating decisions. Negative creditwatch placements lead to an almost 3% decline. After day 15, however, the stock returns increase again. Prior to the final downgrade, the returns are positive, but the downgrade announcement
Creditwatches and their impact on financial markets leads to a sudden price drop. The stock market reaction for positive creditwatch placements is smaller than for negative placements. The positive placements lead to a positive market reaction in the short term, but then the stock returns decline again. This figure shows our first insights into the results. However, it is unable to show the entire evolution from the initial creditwatch placement announcement to the final rating decision, particularly if the credit agency needs a longer time to analyze the information. In the next step, the creditwatch period is standardized to incorporate creditwatch placements with a higher complexity and a longer duration.

Figure 2(a) illustrates graphically the standardized creditwatch process and the standardized cumulative CDS spread changes. We observe that the increase in the abnormal CDS spread changes for firms with a rating placed on creditwatch negative with a subsequent rating downgrade is larger than the decrease for firms with a rating placed on creditwatch positive. During the standardized time period, the cumulative abnormal CDS spread for firms placed on creditwatch negative increases by 55.76 basis points (bps). In contrast, for firms on creditwatch positive that experience a subsequent rating upgrade, there is no observable CDS market reaction until approximately half the standardized time period has passed. During the entire creditwatch procedure, the CDS spread for firms listed on creditwatch positive decreases by 13.17 bps.

Table 3 shows that the CDS spreads increase significantly during the negative creditwatch placement, but no significant decrease is observable during positive creditwatch placements. In addition, no significant equity market reaction can be observed. Figure 1(b) illustrates the normalized abnormal stock returns for firms with a rating placed on creditwatch. Downgrades are slightly negative in the first half of the synthetic trading periods and lead to slightly positive returns in the second period. The downgrade announcement leads to significantly negative returns around the announcement day, but over the entire creditwatch period; the stock performance is not significant. This finding is in stark contrast to Dichev and Piotroski (2001) and Goh and Ederington (1993), who report a significant stock price decline around creditwatch announcements for firms placed on creditwatch negative. The results confirm that, around both announcements, the stock returns significantly decrease for downgrades, but the stock price decline is not significant over the overall period. Positive creditwatch placements are not significant for the CDS market or for the stock market. Longstaff et al (2005) show that approximately 50% of a CDS spread is determined by the default probability of a company. Therefore, the creditwatch is important information for the CDS market. In contrast, Goh and Ederington (1993) show that the equity market reacts differently for rating downgrades: they find that only rating announcements regarding a firm’s financial prospects provide new information to market participants, and other downgrades are already reflected in the stock price. Therefore, the information value of the creditwatch release could vary for the debt and equity markets.
TABLE 3 Creditwatch impact on the CDS and stock markets in the time interval between creditwatch and rating announcement.

<table>
<thead>
<tr>
<th>Creditwatch</th>
<th>n</th>
<th>Mean CASC</th>
<th>Median CASC</th>
<th>CASC &gt; 0 (%)</th>
<th>Mean CAR</th>
<th>Median CAR</th>
<th>CAR &lt; 0 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative</td>
<td>204</td>
<td>55.76**</td>
<td>6.24***</td>
<td>58</td>
<td>-1.59</td>
<td>-1.09</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.025)</td>
<td>(-3.087)</td>
<td></td>
<td>(-0.825)</td>
<td>(-0.824)</td>
<td></td>
</tr>
<tr>
<td>Positive</td>
<td>107</td>
<td>-13.17</td>
<td>-3.39</td>
<td>42</td>
<td>-1.65</td>
<td>-0.37</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.733)</td>
<td>(-2.371)</td>
<td></td>
<td>(-1.070)</td>
<td>(-0.945)</td>
<td></td>
</tr>
</tbody>
</table>

This table shows the results of the CDS spread and stock return performance for the sample of 311 creditwatch announcements throughout the time a rating is on creditwatch, divided into creditwatches for downgrade and upgrade. The CASC and CAR are standardized, following the approach of Malmendier et al. (2016), between the day of the creditwatch announcement and the final rating decision. The mean and median CASC values are shown in basis points, the mean and median CAR values in percent, and these are tested for significance using the parametric t-test and the nonparametric Wilcoxon signed-rank test. The t-value is given in parentheses under the mean values; the Z-score of the Wilcoxon signed-rank test is given in parentheses under the median values. *, ** and *** denote statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Overall, the results indicate that creditwatch announcements do not have an economic impact on equity markets. The CDS spreads, as a measure of risk, show that the firm’s risk significantly increases until the final rating downgrade. Hull et al. (2004) find an anticipation effect for rating downgrades. This effect may be explained by the creditwatch, because participants in the CDS market already adjust their assessment of a firm’s risk during the creditwatch placement.

We now analyze the information value of the announcement of a creditwatch placement in more detail. The creditwatch placements are subdivided into those attributed to performance, those attributed to capital structure and those attributed to market environment.6 Figure 3 shows the CDS and stock markets’ reaction split according to the reason for the creditwatch placement. Table 4 provides the corresponding test statistics. The results indicate that negative creditwatch placements due to performance are highly significant for CDS and stock markets. Negative creditwatch placements due to performance lead to a CDS spread increase of 83.73bps and to an abnormal stock decline of 5.68% over the creditwatch period. Due to regular interactions with the firm’s top management, a CRA receives public and nonpublic information about the current financial status of the firm. For market participants, the firm’s performance is difficult to anticipate. Therefore, a creditwatch placement offers new information and increases the uncertainty as to whether the firm is able to prevent a downgrade.

6 Positive creditwatch placements due to market environment are excluded owing to the small sample size.
Creditwatches and their impact on financial markets

FIGURE 3 Creditwatch cause and the impact on CDS and stock markets’ performance.

(a) CDS spread changes during the creditwatch negative period. (b) CDS spread changes during the spread for creditwatch positive period. (c) Stock market performance during the creditwatch negative period. (d) Stock market performance during the creditwatch positive period. The complete sample consists of 311 creditwatch announcements throughout the time period a rating is on review, divided into reviews for downgrade and upgrade. The results are further divided into operating performance, capital structure and market conditions.

Creditwatch placements attributed to changes in the capital structure are only significant for the CDS market. The CDS spread increase in combination with the positive, albeit not statistically significant, stock return indicates a possible wealth transfer from bondholders to stock holders (Goh and Ederington 1993). Goh and Ederington (1993) argue that rating announcements should be an unexpected surprise for equity investors, and stock prices could also increase when a CRA foresees an increase in a firm’s leverage. This argument follows Holthausen and Leftwich (1986) and Jorion and Zhang (2007). The results in the present paper indicate that creditwatch
TABLE 4  Creditwatch impact on the CDS and stock markets’ performance in the time interval between creditwatch and rating announcement, by creditwatch category.

(a) Creditwatch negative

<table>
<thead>
<tr>
<th>Category</th>
<th>n</th>
<th>Mean CASC</th>
<th>Median CASC</th>
<th>CASC &gt; 0 (%)</th>
<th>Mean CAR</th>
<th>Median CAR</th>
<th>CAR &gt; 0 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td>107</td>
<td>83.73**</td>
<td>7.42***</td>
<td>63</td>
<td>−5.68**</td>
<td>−2.28**</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.009)</td>
<td>(−3.956)</td>
<td></td>
<td>(−2.377)</td>
<td>(−2.104)</td>
<td></td>
</tr>
<tr>
<td>Capital structure</td>
<td>40</td>
<td>41.98**</td>
<td>8.54**</td>
<td>65</td>
<td>2.94</td>
<td>−0.65</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.339)</td>
<td>(−2.567)</td>
<td></td>
<td>(0.692)</td>
<td>(−0.565)</td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>57</td>
<td>12.93</td>
<td>(−19.06)</td>
<td>44</td>
<td>2.91</td>
<td>0.35</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.220)</td>
<td>(−0.520)</td>
<td></td>
<td>(0.689)</td>
<td>(−0.679)</td>
<td></td>
</tr>
</tbody>
</table>

(b) Creditwatch positive

<table>
<thead>
<tr>
<th>Category</th>
<th>n</th>
<th>Mean CASC</th>
<th>Median CASC</th>
<th>CASC &gt; 0 (%)</th>
<th>Mean CAR</th>
<th>Median CAR</th>
<th>CAR &gt; 0 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td>68</td>
<td>−8.04</td>
<td>−6.91**</td>
<td>38</td>
<td>−0.43</td>
<td>0.05</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(−0.504)</td>
<td>(−2.395)</td>
<td></td>
<td>(−0.230)</td>
<td>(−0.214)</td>
<td></td>
</tr>
<tr>
<td>Capital structure</td>
<td>26</td>
<td>4.72</td>
<td>(−0.82)</td>
<td>50</td>
<td>−2.31</td>
<td>0.33</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.096)</td>
<td>(−0.343)</td>
<td></td>
<td>(−0.632)</td>
<td>(−0.419)</td>
<td></td>
</tr>
</tbody>
</table>

This table shows the results of the CDS spread and stock return performance for the sample of 311 creditwatch announcements throughout the time a rating is on creditwatch, divided into creditwatches for downgrade and upgrade and for the creditwatch categories performance, capital structure and market. The CASC and CAR are standardized, following the approach of Malmendier et al (2016), between the day of the creditwatch announcement and the final rating decision. The mean and median CASC values are shown in basis points, the mean and median CAR values in percent, and these are tested for significance using the parametric t-test and the nonparametric Wilcoxon signed-rank test. The t-value is given in parentheses under the mean values; the Z-score of the Wilcoxon signed-rank test is given in parentheses under the median values. *, ** and *** denote statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

placements due to changes in the capital structure are negative signals for bondholders, but equity holders are not affected, and therefore wealth is transformed from bondholders to stock holders.

Negative creditwatch placements due to market conditions are inconclusive. The mean CASC is positive, whereas the median CASC is negative. Neither test statistic supports a significant CDS spread increase or decrease. The stock market reaction is also not statistically significant. Creditwatch placements due to the market environment affect all firms in the same country and industry. In addition, it seems plausible that creditwatch placements and rating downgrades due to the market environment are
not surprising to investors: investors do not require announcements by CRAs during periods of market distress, as they lead the CRAs because the CRAs react only to the negative market environment (Kiesel 2016).

Figure 2 and Table 4 also provide the results for positive creditwatch placements. The impact of positive creditwatch placements is much smaller than that of negative creditwatch placements. The median CASC for creditwatch placements due to firm performance is $-6.91\text{bps}$ and significant at the 5% level. In analogy to negative creditwatch placements, the results indicate that a firm’s performance is difficult to anticipate for market participants, but positive announcements are communicated by the firm’s management rather than by the CRA. These results are in line with the findings of Galil and Soffer (2011). They show that the asymmetry in the CDS and stock markets’ response to rating announcements is associated with asymmetry in contamination clustering.

Finally, we analyze the CDS and stock markets’ reaction on the days of the creditwatch and the final decision announcements. Tables 5 and 6 respectively document the CDS and stock markets’ reactions around the event days. The results are in line with prior research (Goh and Ederington 1993; Hull et al 2004; Norden and Weber 2004). Performance is the only cause for which a significant stock market reaction can be observed, whereas CDS markets also respond to creditwatch placements due to changes in a firm’s capital structure. The results indicate that the CDS market anticipates rating changes due to capital structure and market environment, whereas equity holders are surprised by rating announcements. The CDS and stock markets react more strongly to creditwatch announcements than to the subsequent rating decision, indicating that creditwatch placements provide relevant information to market participants.

5 ROBUSTNESS CHECKS

In order to test the validity of the results achieved so far, multiple robustness checks are applied. First, we use an alternative methodology for calculating abnormal CDS spread changes, to test whether the results are robust to different empirical approaches. Therefore, we use the median spread change of the benchmark CDS index instead of the mean spread change. This approach was used by Galil and Soffer (2011). Using this alternative index calculation procedure minimizes the effect of outliers. Further, we apply another benchmark for stock returns in analogy to the CDS benchmark construction. We adjust the firm’s returns by changes in a stock portfolio of firms in the same rating class as the company’s initial rating.

The results of the CDS market using the median benchmark are presented in Table 7. The robustness test confirms the previous results (see Section 4). The CDS spread increase for firms placed on creditwatch negative is significant and independent of
TABLE 5 CDS market reactions around creditwatch placements and rating changes in the [-2; +2] event window.

(a) Creditwatch negative

<table>
<thead>
<tr>
<th>Category</th>
<th>n</th>
<th>Mean CASC</th>
<th>Median CASC</th>
<th>CASC &gt; 0 (%)</th>
<th>Mean CASC</th>
<th>Median CASC</th>
<th>CASC &gt; 0 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>204</td>
<td>26.60***</td>
<td>5.48***</td>
<td>71</td>
<td>5.91</td>
<td>0.82</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.483)</td>
<td>(–6.612)</td>
<td></td>
<td>(1.512)</td>
<td>(–1.831)</td>
<td></td>
</tr>
<tr>
<td>Performance</td>
<td>107</td>
<td>24.51***</td>
<td>6.46***</td>
<td>76</td>
<td>8.30**</td>
<td>0.74**</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.241)</td>
<td>(–5.859)</td>
<td></td>
<td>(1.995)</td>
<td>(–2.026)</td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>40</td>
<td>22.02***</td>
<td>4.36***</td>
<td>78</td>
<td>2.34</td>
<td>0.92</td>
<td>58</td>
</tr>
<tr>
<td>Structure</td>
<td></td>
<td>(3.621)</td>
<td>(–4.153)</td>
<td></td>
<td>(1.210)</td>
<td>(–0.995)</td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>57</td>
<td>33.75*</td>
<td>4.84*</td>
<td>58</td>
<td>3.91</td>
<td>0.19</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.891)</td>
<td>(–1.680)</td>
<td></td>
<td>(0.338)</td>
<td>(–0.131)</td>
<td></td>
</tr>
</tbody>
</table>

(b) Creditwatch positive

<table>
<thead>
<tr>
<th>Category</th>
<th>n</th>
<th>Mean CASC</th>
<th>Median CASC</th>
<th>CASC &gt; 0 (%)</th>
<th>Mean CASC</th>
<th>Median CASC</th>
<th>CASC &gt; 0 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>107</td>
<td>–4.48</td>
<td>–3.14***</td>
<td>33</td>
<td>2.08</td>
<td>0.10</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(–0.924)</td>
<td>(–3.835)</td>
<td></td>
<td>(1.477)</td>
<td>(–1.032)</td>
<td></td>
</tr>
<tr>
<td>Performance</td>
<td>68</td>
<td>–0.46</td>
<td>–4.71***</td>
<td>28</td>
<td>2.49</td>
<td>0.10</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(–0.066)</td>
<td>(–3.281)</td>
<td></td>
<td>(1.486)</td>
<td>(–1.289)</td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>26</td>
<td>–13.20*</td>
<td>–2.73*</td>
<td>38</td>
<td>0.91</td>
<td>0.23***</td>
<td>58</td>
</tr>
<tr>
<td>structure</td>
<td></td>
<td>(–1.803)</td>
<td>(–1.765)</td>
<td></td>
<td>(0.255)</td>
<td>(–0.292)</td>
<td></td>
</tr>
</tbody>
</table>

This table shows the results of the short-term CDS market reaction for the entire sample of 311 creditwatch placements and their subsequent rating decision, divided into creditwatch placements for downgrade and upgrade. The mean and median CASC are shown in basis points and are tested for significance using the parametric t-test and the nonparametric Wilcoxon signed-rank test. The t-value is given in parentheses under the mean values; the Z-score of the Wilcoxon signed-rank test is given in parentheses under the median values. *, ** and *** denote statistical significance at the 10%, 5%, and 1%, respectively.

The methodology used. At the same time, a tendency for a positive, but not statistically significant, CDS spread decrease for firms placed on creditwatch positive is observable. The results for the different creditwatch categories are in line with the previous results. The strongest effect can be found for creditwatch placements due to firm performance, whereas creditwatches due to the market environment are not statistically significant. Overall, the results for the CDS market reaction are robust to different model specifications.
## TABLE 6  Stock market reactions around creditwatch placements and rating changes in the \([-2: +2]\) event window.

### (a) Creditwatch negative

<table>
<thead>
<tr>
<th>Category</th>
<th>n</th>
<th>Mean CAR</th>
<th>Median CAR</th>
<th>CAR &lt; 0 (%)</th>
<th>Mean CAR</th>
<th>Median CAR</th>
<th>CAR &lt; 0 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>204</td>
<td>-2.77***</td>
<td>-2.24***</td>
<td>64</td>
<td>-1.78***</td>
<td>-0.91***</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-3.120)</td>
<td>(-4.040)</td>
<td></td>
<td>(-3.064)</td>
<td>(-3.704)</td>
<td></td>
</tr>
<tr>
<td>Performance</td>
<td>107</td>
<td>-3.78***</td>
<td>-2.55***</td>
<td>65</td>
<td>-2.03***</td>
<td>-0.97***</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-3.082)</td>
<td>(-3.440)</td>
<td></td>
<td>(-3.248)</td>
<td>(-3.525)</td>
<td></td>
</tr>
<tr>
<td>Capital structure</td>
<td>40</td>
<td>0.16</td>
<td>-1.44</td>
<td>58</td>
<td>-1.62</td>
<td>0.15</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.238)</td>
<td>(-0.804)</td>
<td></td>
<td>(-0.367)</td>
<td>(-0.529)</td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>57</td>
<td>-2.91**</td>
<td>-2.85*</td>
<td>67</td>
<td>-1.42*</td>
<td>-1.35*</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.222)</td>
<td>(-1.886)</td>
<td></td>
<td>(-1.660)</td>
<td>(-1.911)</td>
<td></td>
</tr>
</tbody>
</table>

### (b) Creditwatch positive

<table>
<thead>
<tr>
<th>Category</th>
<th>n</th>
<th>Mean CAR</th>
<th>Median CAR</th>
<th>CAR &lt; 0 (%)</th>
<th>Mean CAR</th>
<th>Median CAR</th>
<th>CAR &lt; 0 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>107</td>
<td>0.51</td>
<td>0.49</td>
<td>47</td>
<td>-0.02</td>
<td>-0.18</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.267)</td>
<td>(0.956)</td>
<td></td>
<td>(-0.196)</td>
<td>(-0.036)</td>
<td></td>
</tr>
<tr>
<td>Performance</td>
<td>68</td>
<td>1.16</td>
<td>0.97</td>
<td>44</td>
<td>-0.30</td>
<td>-0.70</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.550)</td>
<td>(1.235)</td>
<td></td>
<td>(-0.738)</td>
<td>(-0.531)</td>
<td></td>
</tr>
<tr>
<td>Capital structure</td>
<td>26</td>
<td>-0.80</td>
<td>0.12</td>
<td>50</td>
<td>0.57</td>
<td>0.53</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.028)</td>
<td>(0.210)</td>
<td></td>
<td>(0.533)</td>
<td>(1.161)</td>
<td></td>
</tr>
</tbody>
</table>

This table shows the results of the short-term stock market reaction for the entire sample of 311 creditwatch placements and their subsequent rating decision, divided into creditwatch placements for downgrade and upgrade. The mean and median CAR are shown in percent and are tested for significance using the parametric BMP test and the nonparametric CZ test. The Z-score of the BMP is given in parentheses under the mean values; the Z-score of the CZ test is given in parentheses under the median values. *, **, and *** denote statistical significance at the 10%, 5%, and 1%, respectively.

The results for the stock market are also provided in Table 7. The results are similar to the previous results using the national benchmark return. The average cumulative abnormal stock returns (ACAR) are slightly higher in value using the alternative approach. This can be explained by keeping the index the same as before the rating change. Neither the parametric test nor the nonparametric test show different statistical significances. Overall, the CDS and stock markets’ reactions are robust to different model specifications.
This table shows the creditwatch impact on the CDS and stock markets in the time interval between creditwatch and rating announcement using the median spread change of the benchmark CDS index instead of the mean spread change and a risk-adjusted stock benchmark. The CASC and CAR are standardized following the approach of Malmendier et al (2016) between the day of the creditwatch announcement and the final rating decision. The mean and median CASC values are shown in basis points, the mean and median CAR values in percent, and these are tested for significance using the parametric t-test and the nonparametric Wilcoxon signed-rank test. The t-value is given in parentheses under the mean values; the Z-score of the Wilcoxon signed-rank test is given in parentheses under the median values. *, ** and *** denote statistical significance at the 10%, 5%, and 1%, respectively.

Further, as an additional robustness test, we conduct a cross-sectional regression analysis to more closely examine the drivers of the CDS and stock markets’ reactions during the creditwatch procedure. The multivariate ordinary least squares (OLS) regression for abnormal CDS spreads takes the form

\[ CASC_j = \beta_{CASC,0} + \sum_{i=1}^{m} \beta_{CASC,i} \text{VAR}_i + \epsilon. \]  (5.1)

### TABLE 7 Robustness of results.

<table>
<thead>
<tr>
<th></th>
<th>CDS market</th>
<th></th>
<th></th>
<th>Stock market</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>Mean CASC</td>
<td>Median CASC</td>
<td>Mean CASC</td>
<td>Median CASC</td>
<td>Mean CAR</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; 0 (%)</td>
<td></td>
<td>&lt; 0 (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>204</td>
<td>62.20* (1.909)</td>
<td>7.98*** (−3.670)</td>
<td>−3.86* (−1.852)</td>
<td>−1.81 (−1.634)</td>
<td>53</td>
</tr>
<tr>
<td>Performance</td>
<td>107</td>
<td>107.89** (2.178)</td>
<td>11.45*** (−4.556)</td>
<td>−8.62*** (−3.122)</td>
<td>−5.27*** (−2.918)</td>
<td>60</td>
</tr>
<tr>
<td>Capital structure</td>
<td>40</td>
<td>50.03 (2.220)</td>
<td>4.85** (−2.312)</td>
<td>2.55 (0.548)</td>
<td>1.87 (0.565)</td>
<td>45</td>
</tr>
<tr>
<td>Market</td>
<td>57</td>
<td>−15.02 (−0.221)</td>
<td>−7.12 (−0.497)</td>
<td>0.67 (0.163)</td>
<td>1.62 (−0.294)</td>
<td>46</td>
</tr>
</tbody>
</table>

This table shows the creditwatch impact on the CDS and stock markets in the time interval between creditwatch and rating announcement using the median spread change of the benchmark CDS index instead of the mean spread change and a risk-adjusted stock benchmark. The CASC and CAR are standardized following the approach of Malmendier et al (2016) between the day of the creditwatch announcement and the final rating decision. The mean and median CASC values are shown in basis points, the mean and median CAR values in percent, and these are tested for significance using the parametric t-test and the nonparametric Wilcoxon signed-rank test. The t-value is given in parentheses under the mean values; the Z-score of the Wilcoxon signed-rank test is given in parentheses under the median values. *, ** and *** denote statistical significance at the 10%, 5%, and 1%, respectively.
and the multivariate OLS regression for stock returns is specified as

$$\text{CAR}_j = \beta_{\text{CAR},0} + \sum_{i=1}^{m} \beta_{\text{CAR},i} \text{VAR}_i + \epsilon,$$

(5.2)

where CASC\(_j\) is the mean abnormal CDS spread change and CAR\(_j\) is the cumulative abnormal return of firm \(j\) during the standardized creditwatch time period; \(\beta_0\) are the regression constants; \(\beta_i\) are the regression coefficients for the independent variables \(i\); VAR\(_i\) are the independent variables; \(m\) is the number of variables; and \(\epsilon\) is the error term. Standard errors are corrected for heteroscedasticity using robust standard errors.

In addition to the reason for the creditwatch, several firm- and creditwatch-specific variables are used to explain the CASC and CAR during the standardized creditwatch period.

The previous results support the assumption that the CDS and stock markets’ reactions depend on the information value of the creditwatch announcement by the CRA. In order to formally test the relationship between the creditwatch placement and the reason behind it, the variable “Performance” is defined as 1 if the creditwatch process is attributable to firm performance, and 0 otherwise. The variable “Rating” is based on the numerical six-step rating scale (AAA/AA = 6, A = 5, …, CCC and below = 1).

Norden and Weber (2004) find in their cross-sectional analysis that the level of the old rating significantly influences the magnitude of the abnormal performance. The size effects are measured by the logarithm of the market capitalization (MarketCap) on the last trading day of the quarter before the month of the rating review announcement. To examine whether the impact of the creditwatch changed during the financial crisis, the variable “Crisis” is introduced. The global financial crisis peaked between 2007 and mid-2009, so the dummy variable is defined as 1 if the rating event is between December 1, 2007 and June 30, 2009. This time interval is also defined by the National Bureau of Economic Research (NBER) as the recession period due to the global financial crisis. The variable “Net days” is defined as the logarithm of the trading days between the creditwatch announcement and the final rating decision.

Table 8 shows the results of the multivariate cross-sectional regression analysis. These indicate that for negative creditwatch placements the reason for performance is statistically significant at at least the 5% level. This is robust for the CDS and the stock markets. Moreover, the previous rating has a significant impact on the CDS market reaction. The CDS market reaction is stronger for non-investment-grade firms than for firms in a better rating class for positive and negative creditwatch placements. The coefficient “Crisis” shows no significance, indicating that the financial crisis period has no effect on the impact of creditwatches on financial markets. The results indicate no driver for firms placed on creditwatch positive. In summary, the results of the OLS analysis again support that negative creditwatch placements increase the CDS spread, but that this increase depends on the reason for the creditwatch.
TABLE 8  Results of the OLS regression.

<table>
<thead>
<tr>
<th></th>
<th>CDS market</th>
<th>Stock market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Creditwatch negative</td>
<td>Creditwatch positive</td>
</tr>
<tr>
<td>Constant</td>
<td>947.745 (1.259)</td>
<td>182.197 (1.297)</td>
</tr>
<tr>
<td>Performance</td>
<td>120.456** (2.101)</td>
<td>35.472 (0.930)</td>
</tr>
<tr>
<td>Rating</td>
<td>-89.761** (−2.033)</td>
<td>40.132* (1.674)</td>
</tr>
<tr>
<td>MarketCap</td>
<td>16.686 (0.429)</td>
<td>-27.528 (−1.621)</td>
</tr>
<tr>
<td>Crisis</td>
<td>-8.598 (−0.144)</td>
<td>-88.246 (−1.610)</td>
</tr>
<tr>
<td>Net days</td>
<td>-146.282*** (−3.870)</td>
<td>-13.920 (−0.744)</td>
</tr>
<tr>
<td>n</td>
<td>204</td>
<td>107</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.099</td>
<td>0.067</td>
</tr>
</tbody>
</table>

This table displays the coefficients of the driver analysis. The $t$-value is given in parentheses. Standard errors are corrected for heteroscedasticity using robust standard errors. *, ** and *** denote statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

6 CONCLUSION

Prior research suggests that creditwatch announcements have a larger economic impact than the actual rating change. The results of our analysis indicate that this is only a snapshot of the creditwatch impact, as a rating continues to be “on watch” over a certain time period. Stock returns are not significant in response to positive and negative creditwatch placements, whereas CDS spreads continuously increase between a negative creditwatch placement and the subsequent rating downgrade. This is a novel result, and therefore creditwatch placements may potentially explain why prior studies found that CDS markets anticipate rating changes. The strongest effect can be observed for creditwatch placements attributable to firm performance. Rating changes due to changes in a firm’s capital structure and market environment are anticipated by market participants, whereas changes due to firm performance are not.

Hull et al (2004) conclude that creditwatch placements contain significant information for market participants, in particular for the CDS market. Our analysis indicates that the impact of the creditwatch on CDS and stock markets depends on the information it offers to market participants. The market reaction increases for firm-specific performance announcements that may affect the future cashflow development.
of a company. Why CDS and stock markets react differently while a rating is on creditwatch could be a promising avenue for future research.

DECLARATION OF INTEREST

The author reports no conflicts of interest. The author alone is responsible for the content and writing of the paper.

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REFERENCES


Research Paper

Financial distress pre-warning indicators: a case study on Italian listed companies

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ABSTRACT

The reform of the Italian insolvency law in 2005 introduced the troubled debt restructuring (TDR) procedure as a means to restore companies that are in financial distress and avoid potential liquidation. The success of this procedure depends strictly on the timeliness of intervention. Therefore, the availability of a prediction tool appears to be crucial. This paper focuses on the ability of accounting ratios to predict the financial distress status of a firm as defined by the law. Based on a linear discriminant analysis, we formulate the probability of a firm filing for TDR in one year, as well as other quantitative techniques that are intended to monitor financial health. Specifically, we begin with a test of Altman’s $Z$, $Z'$ and $Z''$ scores for bankruptcy on the listed Italian companies that filed for TDR in the period 2005–12. The test results do not completely satisfy the TDR prediction. We then introduce a new score formula based on seven accounting ratios and specific coefficients. Several confirmative analyses are also conducted to validate the predictive accuracy and the generalization power of our score formula.

Keywords: financial distress prediction; troubled debt restructuring (TDR); accounting ratios; Italian listed companies; predictive effectiveness of credit risk.
1 INTRODUCTION AND RESEARCH QUESTIONS

The spread of the 2007–9 global financial crisis has heavily affected the Italian entrepreneurial environment, and an ever growing number of companies have progressively experienced financial distress. At the same time, the volume of global restructured debt showed a significant growth rate. Remarkably, specific provisions on debt restructuring are not included in the Italian Civil Code and the national accounting standards set by the Organismo Italiano di Contabilità (OIC). Recently, several significant legal innovations have emerged. The Law Decree No. 35 of March 14, 2005 (converted into Law No. 80 on May 14, 2005) and the Legislative Decree No. 5 of January 9, 2006 introduced a new tool to manage company crisis: the Article 182-bis restructuring agreements (Di Marzio 2006).

The restructuring agreement is defined within the newly issued Italian accounting standard (OIC 6; see Organismo Italiano di Contabilità 2011) as “an operation whereby the creditor (or group of creditors) grants a concession to the debtor in financial difficulties, such that otherwise it would not have agreed”. This new pre-insolvency proceeding, which somewhat resembles the prepackaged plan of Chapter 11 of the US Bankruptcy Code (Altman et al. 2013), is a hybrid method that occurs partly out-of-court and partly in-court and permits a corporate reorganization. The focus is on the agreements between the debtor and the creditors (Chen et al. 1995) to provide a “fresh start”. This “fresh start” provides the company with new opportunities to operate in the market, including new out-of-court restructuring instruments that prevent the liquidation of the company. In particular, restructuring agreements allow debtors to negotiate new conditions with creditors, including the reduction of residual debt or the rescheduling of the original reimbursement plans. Recently, these instruments have been widely used, especially in negotiations with a group of banks.

The Italian bankruptcy reform appears to closely follow a legislative process similar to that in the United States, even if several differences arise. The main difference is the “cram down” rule. According to this rule, as in US legislation, if a class of creditors votes in favor of the agreement in opposition to one or more classes, the debtor can still seek approval of the restructuring plan based on the process of cram down. In contrast, Article 182-bis does not include this provision, and it charges the Italian Court with formal supervision that includes approving the restructuring plan and

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1 Within the US Bankruptcy Code, Chapter 7 refers to liquidation, whereas Chapter 11 “provides for reorganization, usually involving a corporation or partnership. A Chapter 11 debtor usually proposes a plan of reorganization to keep its business alive and pay creditors over time.” (See http://bit.ly/1OIPkqk.)
verifying its compliance with the law. Therefore, the Italian insolvency law appears generally more rigid than US legislation.

Concerning the definition of financial distress, several authors (Altman 1968; Gilson et al 1990; Wruck 1990; Gilbert et al 1990; John 1993; Johnsen and Melicher 1994; Turetsky and McEwen 2001) have stated that a signal of financial distress is an unexpected decrease in cashflows from continuing operations. This decrease could be followed by a reduction in dividend payments, technical or loan default or troubled debt restructuring (TDR).

We consider TDR to be a warning sign based on liquidity, profitability, leverage, solvency and activity, and firms that face financial distress are eligible to access the TDR procedure in order to avoid potential liquidation. The distinction between TDR and liquidation in the current quantitative models is often aligned to specific modeling requirements and the availability of default and bankruptcy data. Several authors have forecast corporate bankruptcy without distinguishing between liquidation and TDR procedures and have represented them as similar events (Beaver 1966; Ohlson 1980; Zmijewski 1984). Other authors have focused on specifically designed methods to predict financial distress and TDR as a consequence (Gilbert et al 1990; John and Vasudevan 1995; Hill 1996; Turetsky and McEwen 2001).

In this framework, we extend the literature by developing a model to predict financial distress and the consequent TDR request. Therefore, our observations and data collection are focused on the period before the request rather than after its homologation by the court (Beaver 1966; Altman 1968, 2000; Ohlson 1980; Zmijewski 1984).

Basically, this study addresses the following research questions.

(1) Is it possible to estimate the probability that a firm will file for the TDR procedure?

(2) Based on this probability, is it possible to create a simple index that describes the financial equilibrium trend by using historical data?

Thus, the present study specifically contributes to the debate by developing a new model that is able to consider the specific features of TDR procedures in the Italian context.

After testing Altman’s scores on a set of Italian listed companies, we conclude that additional ratios could be involved in order to obtain a more powerful decisional tool. Specifically, our contribution is twofold. In the first phase, a descriptive assessment of the distress phenomenon is conducted by using a multivariate discriminant analysis (MDA). Using the accounting ratios, we obtain the scores of a linear boundary for two groups of firms: “distressed” and “nondistressed”. These labels depend on whether
the TDR procedure has been filed. Then, we estimate a yearly probability that each firm will file for TDR, using Italian data.

Interestingly, the coefficients will be different from those obtained by Altman (1968, 2000) for the same ratios when liquidation is investigated. In fact, Altman’s formula has been used to explore the potential for failure. Instead, TDR is a procedure intended to overcome the financial distress of a company in order to regain its health and “going concern” status. In fact, liquidation does not consider the possibility of a firm that continues to operate. With TDR, however, the intention of the Italian legislator is to let a distressed company go down a legally protected path to restore its financial equilibrium. It follows that a financially distressed company presents different features from a company approaching liquidation or failure, whose equilibrium condition is almost certainly compromised.

For this reason, we focused on the specific features that distinguish financial distress and the companies that request (or are eligible for) TDR. Therefore, in the first phase, by beginning with the original formulation of Altman’s $Z$-score, we sought to create a new function to predict the financial distress status, through a more contemporary analysis in a rapidly changing business environment. To this end, we considered two new accounting ratios to be introduced in the score formula because they have a significant ability to represent the financial equilibrium of a company. In fact, these two indicators, ie, the current ratio and the quick ratio, focus on the financial equilibrium of a company in the short term.

As is well known, many companies that experience financial distress still maintain their equilibrium in terms of their revenues-to-costs ratio and market share. In fact, there are cases in which financial equilibrium is corrupted as a consequence of some difficulties in cashing receivables or to incorrect choices in terms of characteristics of funding and/or duration of payables, debts and loans. In such cases, if the company is able to intervene as soon as financial distress is announced, there is a greater probability of success in restoring equilibrium. Therefore, we expect the inclusion of these two ratios to improve the discriminant power of our score formula. Indeed, such ratios are unanimously considered liquidity ratios, and derive directly from the financial aspect of a company’s equilibrium.

In the second phase, we introduce a novel trend indicator, called the $M$-index, to calculate the predictive effectiveness of TDR probabilities. Our results show that the probability of TDR increases when accounting ratios worsen and companies become distressed. Interestingly, we note that, in contrast, the probability trend of nondistressed companies remains constant.

This study should broadly interest researchers, firms, banks and company advisors. It contributes to the previous literature by developing a new model to help financially distressed companies to make prompt decisions in restoring financial equilibrium.
The paper is organized as follows. In Section 2, we discuss a theoretical framework with which to examine firms in financial distress, with a particular focus on TDR in Italy. Section 3 presents our sample, methodology and results. Section 4 offers some concluding remarks.

2 RESOLUTIONS FOR FINANCIAL DISTRESS: THE TROUBLED DEBT RESTRUCTURING PROCEDURE

There are several definitions of the term “financial distress” that have evolved in the literature. Gilson et al (1990) and Wruck (1990) state that a company is in financial distress when its cashflow is insufficient to meet its current liabilities, and this is often resolved through a private workout or legal reorganization under Chapter 11 of the US bankruptcy code. John (1993) describes distress events as points in time when a firm’s liquid assets are insufficient to meet the currency requirements of its contracts. Gilbert et al (1990) and Johnsen and Melicher (1994) compared financial distress with the physical status of a company’s health and suggested that there are heterogeneous financial distress characteristics associated with events between corporate health and bankruptcy. Turetsky and McEwen (2001) described financial distress as a series of financial events that reflect various stages of corporate adversity. Common to all these definitions is the unexpected decrease in cashflow from continuing operations signaling the onset of financial distress, where firms may use the TDR procedure.

According to the TDR literature, a firm that experiences financial distress can use several methods to overcome it, such as voluntary restructuring of its operations (Donaldson 1990; John et al 1992), restructuring under the protection of the bankruptcy court (Weiss 1990) and private restructuring (Gilson et al 1990). Brown (1989), Giammarino (1989) and Mooradian (1994) state that firms resolve their financial distress under Chapter 11, despite incurring additional bankruptcy costs. Under Chapter 11, restructuring is preferable to a workout plan when there are a large number of trade creditors (Gilson et al 1990). The presence of bank debt is positively associated with the success of a workout plan. Chatterjee et al (1996) state that firms filing for Chapter 11 reorganization have more trade credit than those that attempt workout plans. Several authors (Baird 1986; Bebchuk 1988, 2000; White 1989, 1994; Bradley and Rosenzweig 1991; Jensen 1991; Aghion et al 1992; Kaiser 1996) contend that Chapter 11 is a debtor-friendly process that grants controlling power to management and fails to liquidate a significant number of economically inefficient firms.

In contrast, Aivazian and Zhou (2012) suggest that Chapter 11 reorganizations tend to boost the operating performances of firms that face temporary profitability
problems, and they challenge the contention that Chapter 11 is an inefficient debtor-friendly mechanism. Zhang (2010) states that, with protection from Chapter 11, a firm may emerge as a new entity that fiercely competes with its industry rivals.

One view that most of the above-mentioned works have in common is that companies that decide to access TDR procedures are in financial distress; this certainly represents an adverse situation but is distinct from liquidation or failure. Several authors have developed models to forecast potential business failure in the Chapter 11 framework (see, for example, Beaver 1966; Altman 1968, 2000; Ohlson 1980; Zmijewski 1984), and the definition of default in the quantitative literature is often aligned to specific modeling requirements and the availability of default and bankruptcy data. For example, in Zmijewski (1984), financial distress is considered the act of filing a petition for bankruptcy.

Gilson (1990) defined a firm as financially distressed when it is in default on its debt, bankrupt or privately restructuring its debt to avoid liquidation. Ohlson (1980) classified “failed firms” as companies that must file for bankruptcy, including Chapter 10, Chapter 11 and other bankruptcy proceedings. For both these authors, it seems evident that financial distress and liquidation or failure are considered similar. In the literature, Chapter 11 and liquidation proceedings are considered similar events.

Few authors focus on the methods specifically designed to predict financial distress and the consequent filing for the TDR procedure. John and Vasudevan (1995) create a model to predict when “good” and “medium” quality liquid firms may restructure their debt out of court, when “good” quality illiquid firms may use a prepackaged solution, and when “lower” quality (liquid and illiquid) firms may file for Chapter 11. They show that the choice of restructuring method depends on the quality and liquidity of the firm. Hill (1996) emphasizes that an analysis of the resolution of financially distressed firms demands a dynamic methodology because the movement to and from financial distress is a dynamic process. Previous studies of distressed firms have tended to use ex post sampling techniques that are not representative of the general population of financially troubled firms (Gilbert et al. 1990). Alternatively, Turetsky and McEwen (2001) group financially troubled firms according to an initial signal, the decrease in operating cashflows, and track them across various distress points. They incorporate the techniques of survival analysis to examine firm longevity. Survival analysis longitudinally tracks firms after a decline in health, via the subsequent occurrence of dividend reduction, default or TDR.

We notice a clearer distinction from a legislative perspective. In US legislation, Chapter 11 is a company reorganization procedure that focuses on the stipulation of an agreement between debtors and creditors to restore the firm after financial distress.
On the other hand, Chapter 7 exclusively concerns liquidation.² We note that in Italian insolvency law, this distinction also appears in Article 182-­bis and Article 216 et seq., respectively. In fact, the TDR designation (Article 182-­bis) was inspired by the US Bankruptcy Code (namely, Chapter 11).

Several studies have tested the accuracy of Altman’s Z-­score model and its revised versions Z° and Z” (Altman 1993, 2000) outside the United States and across different industries (Hayes et al 2010; Alareeni and Branson 2013).

Some of these studies have confirmed the Z-­score’s validity and prediction ability. Other studies have raised doubts over its applicability to other contexts and emphasized the need to update it according to a new time period and different industries (Begley et al 1996; Lifschutz and Jacobi 2010). Åstebro and Winter (2012) argue that the outcome of financial distress should be modeled using a multinomial specification that distinguishes between failure, survival as going concern and acquisition, rather than using a binary model. Moreover, De Andrés et al (2012) and Tinoco and Wilson (2013) developed risk models for listed companies that try to predict financial distress and bankruptcy by replacing traditional accounting ratios with a combination of accounting data, stock market information and proxies for changes in the macroeconomic environment.

To the best of our knowledge, there are no prior studies on the Italian context except those of Celli (2015) and Altman et al (2013).

Celli (2015) assesses the ability of the Z-­score model to predict the default of a sample of Italian listed firms up to three years beforehand. He concludes that the Z-­score works effectively and performs well in predicting the failures of Italian firms, although with a slightly lower degree of reliability in respect to its application to US and UK companies. Celli considered a simple permanent suspension from quotation, rather than the formal access to a bankruptcy law procedure, as a useful and appropriate proxy for business failure.

Instead, Altman et al (2013) focused on adopting Altman’s Z-­score for Italian companies subject to extraordinary administration. In this case, they expressed concerns regarding the model’s consistency with the specific features of the Italian environment (Altman et al 2013, p. 135).

Unlike Celli (2015), and considering the concerns of Altman et al (2013), our paper shows that applying Altman’s Z-­score model to our main and control samples of Italian companies gives results that are not entirely accurate. This inaccuracy mainly emerges because the conditions of financial distress of the companies that file for (or are eligible to file for) the TDR procedure are different from those of companies that are approaching liquidation or failure. The Italian entrepreneurial context specifically

² Chapter 7 of the Bankruptcy Code provides for “liquidation – the sale of a debtor’s nonexempt property and the distribution of the proceeds to creditors” (see http://bit.ly/1GBfXot).
highlights the prevalent restriction of the ownership of companies and the relevant role of banks in financing companies.

3 DATA AND EMPIRICAL ANALYSIS

3.1 Sample description

Our data, coming from the Analisi Informatizzata delle Aziende Italiane–Bureau Van Dijk database, consists of a panel of fifty companies whose financial reports are made under the International Financial Reporting Standards.

These companies have been labeled “distressed” (or “nondistressed”) if they have filed for Article 182-bis restructuring agreements (or not). The group of “distressed” firms contains all twenty of the listed Italian companies that filed for the TDR procedure from right after the reform (in 2005) up to 2012. The control group comprises thirty companies of similar size and industry type to the distressed companies. Although the control/case ratio of the sample sizes is traditionally taken as equal to 1, we have set it to be 1.5 because the recent literature suggests increasing it improves its convenience.

Although the reform was effective from 2005, we collected financial data from 2003 to 2012 to observe the trend of companies’ ratios before and after the reform.

We computed several accounting ratios for 2003–12. These are denoted by $X_i$, $i = 1, \ldots, 7$:

- $X_1$ denotes a measure of the net liquid assets of the firm, which is calculated as working capital/total assets;
- $X_2$ represents profitability through retained earnings/total assets;
- $X_3$ measures leverage and indicates earnings before interest and taxes (EBIT)/total assets;
- $X_4$ shows the solvency of a firm that involves the market value of equity/book value of total liabilities;
- $X_5$ represents a standard financial ratio that shows sales/total assets;
- $X_6$ is the current ratio between current assets (cash, cash equivalents, short-term receivables and inventory) and short-term liabilities (payables);
- $X_7$ is the quick ratio between total liquidity (cash, cash equivalents and short-term receivables) and short-term liabilities (payables).

The longitudinal nature of our approach is motivated by the fact that filing for the TDR procedure is viewed not as a single instantaneous occurrence but an ongoing
TABLE 1  Industry classification for the sample.

<table>
<thead>
<tr>
<th>Industry type</th>
<th>Number of firms</th>
<th>Distressed (group 1)</th>
<th>Nondistressed (group 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer goods</td>
<td>11</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Consumer service</td>
<td>9</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Finance</td>
<td>7</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Health and food</td>
<td>7</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>5</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Oil and natural gas</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Technology</td>
<td>7</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>50</strong></td>
<td><strong>20</strong></td>
<td><strong>30</strong></td>
</tr>
</tbody>
</table>

The “distressed” group comprises the firms that filed for TDR in 2005–12. The “nondistressed” group comprises the firms selected by following a stratified sampling scheme (by size and industry), and they are matched with the distressed firms in the same observation period for every industry.

process, which evolves over a considerable period of time. We assume, moreover, that this process provides signals that allow the forecasting of future financial TDR filings.

Table 1 summarizes the data by industry classification. Although the macro-industry “health and food” shows no distressed firms, we include this sector in order to consider all the macro-industries on the Milan Stock Exchange, which are subject to the same TDR law (banks and insurance companies are excluded because they are subject to different restructuring and bankruptcy laws).

3.2 Altman’s $Z$-scores of the Italian sample: main results

By adopting Altman’s $Z$-score and its revised models ($Z’$ and $Z’’$) for the above-described Italian sample, we obtain the following results. As represented in Table 2, more than 50% of the control sample companies were classified between the distress zone and the gray zone during the observation period. In the distressed group, the companies always seemed to belong to the distress zone during the observation period, with a significant percentage between 75% and 90%.

Then, we test the $Z’$-score for both groups. As shown in Table 3, on average, only 5.3% of “healthy” firms belong to the safe zone, and this emphasizes that most of these firms are classified between the gray and the distress zones. On the contrary, the distressed sample demonstrates the same state of health (distressed) from the first to the last year of observation. Therefore, the findings from the $Z’$-score do not seem to reflect the natural evolution of the health status of the companies.

The $Z’’$-score should better fit the Italian entrepreneurial context because it is characterized by a stronger relationship between companies and banks. Table 4 shows that, on average, 97% of the distressed companies are classified as being in the distress
TABLE 2  Results of the $Z$-score for the two groups of Italian listed firms.

<table>
<thead>
<tr>
<th>Year</th>
<th>Distressed sample</th>
<th></th>
<th></th>
<th>Nondistressed sample</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Insolvency area</td>
<td>Gray area</td>
<td>Low-risk area</td>
<td>Insolvency area</td>
<td>Gray area</td>
<td>Low-risk area</td>
</tr>
<tr>
<td>2012</td>
<td>90</td>
<td>10</td>
<td>0</td>
<td>50</td>
<td>37</td>
<td>13</td>
</tr>
<tr>
<td>2011</td>
<td>90</td>
<td>10</td>
<td>0</td>
<td>50</td>
<td>33</td>
<td>17</td>
</tr>
<tr>
<td>2010</td>
<td>90</td>
<td>10</td>
<td>0</td>
<td>50</td>
<td>43</td>
<td>13</td>
</tr>
<tr>
<td>2009</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>57</td>
<td>30</td>
<td>13</td>
</tr>
<tr>
<td>2008</td>
<td>95</td>
<td>5</td>
<td>0</td>
<td>53</td>
<td>37</td>
<td>10</td>
</tr>
<tr>
<td>2007</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>47</td>
<td>47</td>
<td>7</td>
</tr>
<tr>
<td>2006</td>
<td>90</td>
<td>5</td>
<td>5</td>
<td>50</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>2005</td>
<td>75</td>
<td>25</td>
<td>0</td>
<td>53</td>
<td>37</td>
<td>10</td>
</tr>
<tr>
<td>2004</td>
<td>75</td>
<td>15</td>
<td>10</td>
<td>43</td>
<td>47</td>
<td>10</td>
</tr>
<tr>
<td>2003</td>
<td>75</td>
<td>15</td>
<td>10</td>
<td>50</td>
<td>37</td>
<td>13</td>
</tr>
</tbody>
</table>


TABLE 3  Results of the $Z'$-score for the two groups of Italian listed firms.

<table>
<thead>
<tr>
<th>Year</th>
<th>Distressed sample</th>
<th></th>
<th></th>
<th>Nondistressed sample</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Insolvency area</td>
<td>Gray area</td>
<td>Low-risk area</td>
<td>Insolvency area</td>
<td>Gray area</td>
<td>Low-risk area</td>
</tr>
<tr>
<td>2012</td>
<td>80</td>
<td>10</td>
<td>10</td>
<td>37</td>
<td>53</td>
<td>10</td>
</tr>
<tr>
<td>2011</td>
<td>85</td>
<td>10</td>
<td>5</td>
<td>33</td>
<td>57</td>
<td>10</td>
</tr>
<tr>
<td>2010</td>
<td>80</td>
<td>20</td>
<td>0</td>
<td>33</td>
<td>60</td>
<td>7</td>
</tr>
<tr>
<td>2009</td>
<td>90</td>
<td>10</td>
<td>0</td>
<td>37</td>
<td>60</td>
<td>3</td>
</tr>
<tr>
<td>2008</td>
<td>90</td>
<td>10</td>
<td>0</td>
<td>30</td>
<td>67</td>
<td>3</td>
</tr>
<tr>
<td>2007</td>
<td>70</td>
<td>30</td>
<td>0</td>
<td>30</td>
<td>67</td>
<td>3</td>
</tr>
<tr>
<td>2006</td>
<td>70</td>
<td>30</td>
<td>0</td>
<td>27</td>
<td>73</td>
<td>0</td>
</tr>
<tr>
<td>2005</td>
<td>55</td>
<td>45</td>
<td>0</td>
<td>40</td>
<td>53</td>
<td>7</td>
</tr>
<tr>
<td>2004</td>
<td>45</td>
<td>50</td>
<td>5</td>
<td>27</td>
<td>67</td>
<td>7</td>
</tr>
<tr>
<td>2003</td>
<td>45</td>
<td>50</td>
<td>5</td>
<td>30</td>
<td>67</td>
<td>3</td>
</tr>
</tbody>
</table>

All values given in percent. Insolvency area: $Z < 1.23$. Gray area: $1.23 < Z < 2.90$. Low-risk area: $Z > 2.90$.

zone from 2003 onward, and it follows that these companies were constantly distressed during the entire observation period. In contrast, more than 80% of the control sample belongs in the distressed area. Therefore, the $Z''$-score does not appear to be useful to describe the health trend of Italian companies from 2003 to 2012.
We emphasize that the results for both the listed Italian companies that filed for the TDR procedure beginning in 2006 and those for the control sample companies are not consistent with their real conditions. In fact, most companies in the distressed group are classified in the insolvency area from 2003 (75% of the sample), and this is inconsistent with the fact that they did not file for bankruptcy until 2012. At the same time, nearly all companies do not belong in the healthy area. In the control sample, the companies are divided between the three areas in a way that does not appear to be very intuitive.

### 3.3 Methodology

Our statistical analysis is divided into two steps. In the first step, we derive two discriminant functions through multivariate linear discriminant analysis: the first function is based on the five original accounting ratios \(X_1, \ldots, X_5\) adopted by Altman (1968, 1993); the second function is based on the five original accounting ratios plus the current ratio \(X_6\) and the quick ratio \(X_7\).

In the second step, we use the scores of the linear discriminant analysis to estimate the yearly degree of confidence of the likelihood of each firm resorting to Article 182-bis restructuring agreements in the following year. This assessment method appears to be fairly novel in the field of financial distress.

A confirmative analysis is included to assess the predictive accuracy of our likelihood measurement. To this end, we introduce a simple measure, called the \(M\)-index.
which is calculated for each distressed firm. This measure is obtained based on the estimated TDR probabilities calculated right before the TDR request. This index should signal the likelihood of financial conditions worsening, and it could also be considered a predictive tool for the TDR request procedure.

### 3.4 Linear discriminant analysis

Accounting ratios have been widely adopted as a method of determining the relative strength and performance of companies for financial statement analysis. Ratio analysis helps to identify trends over time for a company and to compare two or more companies at a single point in time. Ratios usually focus on three key aspects of a business: liquidity, profitability and solvency. The reliability of this type of analysis lies in the reliability of the reported accounting numbers. Therefore, if fraudulent policies are adopted in financial reporting, the accounting ratios are misleading. Nevertheless, we assume that the financial reports of the listed Italian companies that are included in the present observation are reliable, because they have been reviewed by auditing firms and by the Commissione Nazionale per le Società e la Borsa (the authority that regulates the Italian financial market).

To conduct a discriminant analysis, we first use the five financial ratios suggested by Altman (1968, 1993), which are specific to liquidation/failure studies. Then, we run the discriminant analysis by adding to these the current ratio ($X_6$) and the quick ratio ($X_7$) with the aim of increasing the detecting power of our score formula in terms of the financial equilibrium of the company. In fact, the general equilibrium of a company is based on an unstable combination of economic equilibrium (based on profitability) and financial equilibrium (based on liquidity and solvency). $X_6$ and $X_7$ are typically considered to be liquidity ratios because they measure the ability of a company to repay short-term debts and meet unexpected cash needs. Provided that a deterioration in this ability eventually influences the profitability of the company, a sudden intervention could often prevent the deterioration in economic equilibrium. The TDR procedure is fit for this purpose because it could be helpful to restore the financial equilibrium when the company is still viable and sound from a profitability point of view. For these reasons, we assume that these variables play a key role in explaining financial distress. In our specific case, these ratios should have a high discriminant power for the prediction of filing for Article 182-\textit{bis} restructuring agreements. In our study, the companies that filed for TDR are labeled “distressed”, whereas the control sample companies are labeled “nondistressed”.

The number of misclassified observations is reported in Table 5(a) for the five original accounting ratios and in Table 5(b) for the score with the two additional ratios.
We note a reasonable discriminant power in both analyses, but, as expected, this performs better if we consider the seven accounting ratios (Table 5(b)), especially in recent years. This result confirms that the “distressed” labels mainly assume significance with the imminence of a request for TDR. Moreover, the errors reduce over the years, because when data of a restructured firm dates back sufficiently far from the TDR recourse, the “distressed” status makes little sense and generates a widely biased boundary.

In order to check for the absence of overfitting, we used tenfold cross-validation. This scheme requires, at each of ten steps, the prediction of 10% of data by using the remaining 90%. This has been done for each year, and Table 6 shows the average error rates for the ten predictions. It can be seen that the prediction error rates are quite similar to the misclassification rates over time, as shown in Table 5(b). By contrast, overfitting would have typically showed high performance in some years and poor performance in other years.

3.5 Developing a TDR probability model: $S_{it}$ score

In the second step of our statistical analysis, we assess the probability of financially distressed companies filing for TDR as a way to avoid failure. This is estimated via the explanatory variables. The research closest to our approach is by Shumway (2001), who also calculates the probability of a business failure event by using a logistic regression model. In addition, this approach relates to the previous literature pioneered by Beaver (1966) and Altman (1968, 1993, 2000), who introduced several models as measures of bankruptcy risk. The well-known studies by these authors investigate the ability of financial ratios to predict corporate financial distress. Beaver’s (1966) study notes that financial ratios can predict the likelihood of bankruptcy. In particular, he confirmed that these financial ratios can provide significant information and warning about the financial conditions of firms before their liquidation. Subsequently, Altman (1968) developed a model that included a set of financial ratios that were analyzed through MDA to demonstrate the relation between the financial ratios in previous years and at the time of the subsequent bankruptcy.

Our formula for the TDR probability associated to the $i$th firm at time $t$ is as follows:

$$\hat{P}_{it} = \frac{e^{S_{it-1}}}{1 + e^{S_{it-1}}},$$

(3.1)

where $e$ indicates the Euler number, and $S_{it}$ is a score associated with the $i$th firm at time $t$, which is given by the following formula:

$$S_{it} = 0.003X_{1it} + 0.267X_{2it} + 0.663X_{3it} + 0.431X_{4it} + 0.533X_{5it} + 0.147X_{6it} + 0.092X_{7it},$$

(3.2)

where $X_{jit}$ is the $j$th variable associated with the $i$th firm at time $t$. 

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TABLE 5  Multivariate discriminant analysis with (a) five accounting ratios and (b) seven accounting ratios.

<table>
<thead>
<tr>
<th></th>
<th>Distressed firms</th>
<th>Nondistressed firms</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>8 (40%)</td>
<td>6 (20%)</td>
<td>12</td>
</tr>
<tr>
<td>2004</td>
<td>7 (35%)</td>
<td>5 (16%)</td>
<td>12</td>
</tr>
<tr>
<td>2005</td>
<td>7 (35%)</td>
<td>8 (26%)</td>
<td>15</td>
</tr>
<tr>
<td>2006</td>
<td>3 (15%)</td>
<td>10 (33%)</td>
<td>13</td>
</tr>
<tr>
<td>2007</td>
<td>3 (15%)</td>
<td>6 (20%)</td>
<td>9</td>
</tr>
<tr>
<td>2008</td>
<td>1 (3%)</td>
<td>6 (20%)</td>
<td>7</td>
</tr>
<tr>
<td>2009</td>
<td>2 (10%)</td>
<td>1 (3%)</td>
<td>5</td>
</tr>
<tr>
<td>2010</td>
<td>3 (15%)</td>
<td>7 (23%)</td>
<td>10</td>
</tr>
<tr>
<td>2011</td>
<td>3 (15%)</td>
<td>5 (16%)</td>
<td>8</td>
</tr>
<tr>
<td>2012</td>
<td>7 (35%)</td>
<td>5 (16%)</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Distressed firms</th>
<th>Nondistressed firms</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>6 (32%)</td>
<td>4 (14%)</td>
<td>10</td>
</tr>
<tr>
<td>2004</td>
<td>7 (37%)</td>
<td>6 (20%)</td>
<td>13</td>
</tr>
<tr>
<td>2005</td>
<td>6 (32%)</td>
<td>9 (30%)</td>
<td>15</td>
</tr>
<tr>
<td>2006</td>
<td>3 (15%)</td>
<td>10 (34%)</td>
<td>13</td>
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<tr>
<td>2007</td>
<td>3 (15%)</td>
<td>5 (16%)</td>
<td>8</td>
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<tr>
<td>2008</td>
<td>2 (10%)</td>
<td>7 (23%)</td>
<td>9</td>
</tr>
<tr>
<td>2009</td>
<td>1 (5%)</td>
<td>2 (6%)</td>
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<td>3 (10%)</td>
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<td>3 (15%)</td>
<td>6 (20%)</td>
<td>9</td>
</tr>
<tr>
<td>2012</td>
<td>7 (37%)</td>
<td>2 (6%)</td>
<td>8</td>
</tr>
</tbody>
</table>

Part (a) shows the number of misclassified firms of the linear discriminant analysis from 2003–12 for the distressed and nondistressed groups based on the five original accounting ratios (Altman 1968, 1993). Part (b) shows the number of misclassified firms of the linear discriminant analysis from 2003–12 for the distressed and nondistressed groups based on the seven accounting ratios. The figures between parentheses represent percentages of error.

The value of $\hat{P}_{it}$ is always between 0 and 1 and quantifies the opportunity for a firm to file for TDR. As can clearly be seen from the above formula, given that $S_{it}$ is inversely proportional to the firm’s health status, this probability increases with the tendency to file for TDR.
TABLE 6  Averages of prediction error rates using tenfold cross-validation.

<table>
<thead>
<tr>
<th>Year</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>26</td>
</tr>
<tr>
<td>2004</td>
<td>24</td>
</tr>
<tr>
<td>2005</td>
<td>34</td>
</tr>
<tr>
<td>2006</td>
<td>24</td>
</tr>
<tr>
<td>2007</td>
<td>18</td>
</tr>
<tr>
<td>2008</td>
<td>22</td>
</tr>
<tr>
<td>2009</td>
<td>14</td>
</tr>
<tr>
<td>2010</td>
<td>16</td>
</tr>
<tr>
<td>2011</td>
<td>16</td>
</tr>
<tr>
<td>2012</td>
<td>14</td>
</tr>
</tbody>
</table>

This table shows the results of a tenfold cross-validation in the form of the average error rates over the ten predictions.

The numerical coefficients in the above formula have been obtained from a linear discriminant analysis, where the ratios used for each firm are as follows:

(a) for distressed firms, the ratios belong to the year when the TDR was filed;

(b) for nondistressed firms, the ratios are the best ratios over the entire observation period.

This approach allows an efficient estimation of the $S_{it}$ score for both the high discriminant potential of the ratios chosen in this manner and the reduction in temporal correlation because the data from different years are compared. A fundamental assumption for our multivariate estimation is the independence of sample observations, which has rarely been considered seriously in prior studies.

Comparing the $S_{it}$ score formula with the $Z$-score (Altman 1968, 1993, 2000), it appears that the same ratios can exert different influences on the formula because of the different coefficients. This effect may be explained by noting that financial distress and liquidation/failure are different phenomenons, because these ratios may affect a company’s choice to access TDR procedures without necessarily resulting in liquidation. Moreover, the $Z$-score formula comes from data collected over a one-year interval that is focused only on manufacturing companies, whereas $S_{it}$ is obtained from a multi-year and multi-industry analysis of listed Italian companies. Further, the $S_{it}$ score is calculated to be included in the determination of $\hat{P}_{it}$, which represents the assessment of the probability of TDR for a company. On the contrary, the $Z$-score formula was developed to determine a cut-off value.
Predictive effectiveness of the TDR probabilities for both the distressed and nondistressed groups.

3.6 The effectiveness of TDR probabilities

In this section, we analyze the predictive accuracy of the TDR probabilities that were introduced in the previous section. Assuming that the TDR request is beneficial in overcoming a temporary state of crisis, the analysis of the time series \( \hat{P}_{it}, t = 2003; \ldots; 2012 \), should satisfy the following two hypotheses.

(H1) For “distressed” companies, the associated probability should increase until the year of TDR application and then become stationary or even decrease.

(H2) For “nondistressed” companies, the time series should appear stationary and follow a nearly constant trend over the years.

We investigate these hypotheses below.

First, we introduce an indicator, the \( M \)-index, which is intended to determine whether, for a distressed firm, a relative increment in TDR probability is expected to occur in the proximity of the request for TDR. This indicator is given by the following expression:

\[
M_i = \frac{\max(\hat{P}_{i,t_0}, \hat{P}_{i,t_0-1}, \hat{P}_{i,t_0-2})}{\sum_{t=t_{\min}}^{t_{\max}} \hat{P}_{i,t}/T}.
\] (3.3)

The above index, which is calculated for the \( i \)th distressed firm, is constructed from a ratio whose denominator is the average of the TDR probabilities over the observational period. The numerator is calculated as the maximum probability associated with the two years before the restructuring agreements and the year when TDR was effectively approved in court (the latter is denoted by \( t_0 \)). The symbols \( t_{\min} \) and \( t_{\max} \) represent the first and last observation years in the analysis. Clearly, the more the probabilities increase before a TDR request, the more likely it is that the indicator is greater than 1. Moreover, we selected the years of the best performing ratios for the control sample
companies, given that these companies did not file for TDR during the observation period. For these years, we calculated the respective probabilities in order to obtain the $M$ -index for the control sample companies. Therefore, the black bars in Figure 1 show the values of the $M$ -index of the distressed sample (the period considered for calculating the denominator is from 2003 to 2012). This result clearly confirms the strong predictive potential of TDR probabilities in our case because for the distressed firms this indicator is greater than 1 as the probabilities increase before the TDR request. The gray bars represent the values of the $M$ -index of the control sample companies. On average, the gray bars are almost always below 1.

Although the $M$ -index is a validation tool that focuses on a single firm, a general view could be gained by calculating the trends of the average of the TDR probabilities for the two groups of firms for every year.

In Figure 2, we represent the two trends. In particular, the solid line indicates the trend of the distressed firms. For a simpler comparison between the two groups, we have not used standardized values.

We note that the trend of the distressed firms increases with increasing proximity to the critical period. Interestingly, we have sixteen TDR requests in the last three years (from a total of twenty cases), which is when the trend approached its maximum. In contrast, the dashed line represents the average of the probabilities of nondistressed firms to file for TDR. In this case, we have a nearly constant trend, which provides no cause for concern.

Table 7 shows that the values of the averages of the case and control groups differ significantly for each year from 2006 onward at the usual smallest levels of confidence. This shows that it is possible to generalize our result to the potential population of firms with the same features as those of the distressed sample.
TABLE 7  Comparison between TDR probability series.

<table>
<thead>
<tr>
<th>Year</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>0.3113145</td>
</tr>
<tr>
<td>2004</td>
<td>0.100861</td>
</tr>
<tr>
<td>2005</td>
<td>0.1034408</td>
</tr>
<tr>
<td>2006</td>
<td>0.0003056672</td>
</tr>
<tr>
<td>2007</td>
<td>0.0005674411</td>
</tr>
<tr>
<td>2008</td>
<td>0.002576639</td>
</tr>
<tr>
<td>2009</td>
<td>0.0001212181</td>
</tr>
<tr>
<td>2010</td>
<td>0.0002130407</td>
</tr>
<tr>
<td>2011</td>
<td>0.00003964437</td>
</tr>
<tr>
<td>2012</td>
<td>0.001247192</td>
</tr>
</tbody>
</table>

p-values related to two sample t-tests comparing average TDR probabilities of distressed and control firms between 2003 and 2012.

FIGURE 3  Sequence of distributions of TDR probabilities, distressed firms.

In order to show the model accuracy at the firm level, in Figure 3 we represent the $\hat{P}_{i,t}$ data using box plots. With regard to the distressed companies (Figure 3), the increasing trend in the medians and the decently concentrated distributions (except for 2012), along with a moderate number of outliers, show that our analysis is relatively robust to the presence of false positives. The correspondence between the medians for
2011 and 2012 suggests stationarity, as some companies that previously filed for TDR start to restore financial equilibrium and therefore have slightly decreased their $\hat{P}_{IT}$. However, for our control sample companies (Figure 4), the trend of medians appears to be stationary, which confirms the robustness of the model at the firm level.

### 4 SUMMARY AND CONCLUSIONS

In this paper, we describe the ability of accounting ratios to forecast the probability of a financially distressed firm filing for TDR. To this end, we constructed a financial “prewarning” model to predict the financial distress of listed companies. In this context, we collected the financial data from 2003 to 2012 for all twenty listed Italian companies that filed for Article 182-bis restructuring agreements and for a control group of thirty other companies listed on the Milan Stock Exchange. Our data set is a panel covering a period of nine years and is not considered small.

Using the MDA, we created a simple and efficient discriminant function. Once assured that the seven ratios we adopted are informative regarding financial distress, we defined the probability of a TDR filing. Based on this probability, we also introduced a user-friendly indicator, the $M$-index, which can describe the financial equilibrium trend using historical data.

As a result of our findings, we suggest the application of this model to predict the financial distress of firms across different industries.
This study provides tools that could be useful not only to companies undergoing reorganization but also to banks, creditors and investors, which can evaluate the opportunity to suggest TDR at the correct time.

Further research could assess the efficiency of the TDR procedure in pursuing the legislator’s aims in reforming the bankruptcy law, ie, a reduction in the company failure rate during periods of financial crisis.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

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Research Paper

Stochastic loss given default and exposure at default in a structural model of portfolio credit risk

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ABSTRACT

In this paper, we develop a factor-type latent variable model for portfolio credit risk that accounts for stochastically dependent probability of default (PD), loss given default (LGD) and exposure at default (EAD) at both the systematic and borrower-specific levels. By employing a comprehensive simulation study, we set our results in contrast to those obtained using the asymptotic single risk factor (ASRF) model that underlies Basel II and III. Several sensitivity and robustness analyses for different parameter assumptions are conducted to break down our results. As required by the regulator, we show how to map our portfolio credit loss quantiles with correlated PD, LGD and EAD into values for downturn LGD and EAD. Our analyses reveal that stochastically dependent defaults, LGD and EAD increase a credit portfolio’s tail risk significantly. Estimating risk markups separately can therefore lead to substantially underestimating the inherent risk of a portfolio with nondeterministic exposures.
Further, we show that relative downturn markups on LGD and EAD depend strongly on asset correlations and, in particular, that credit portfolios with low asset correlations might be prone to an underestimation of additional capital charges for stochastically dependent LGD and EAD. Thus, our results are of economic significance for banks and regulators when setting up minimum capital requirements.

**Keywords:** portfolio credit risk model; asymptotic single risk factor (ASRF) model; downturn loss given default (LGD); downturn exposure at default (EAD); correlation structure.

## 1 INTRODUCTION

Modeling portfolio credit risk and, in particular, default dependencies using Merton-type latent variable models, where the dependency between the latent variables is defined by a factor approach, has grown in prevalence since the introduction and development of the asymptotic single risk factor (ASRF) model by Vasicek (1987, 2002), Gordy (2000, 2003) and others.\(^1\) The ASRF model provides the basis for banks to determine their minimum regulatory capital requirements for credit risk when adopting the internal ratings-based approach (IRBA) within the Basel II and III frameworks.\(^2\)

In the Basel framework, as well as in internal credit risk models, the major elements driving a bank’s credit portfolio risk are a borrower’s probability of default (PD), loss given default (LGD) and exposure at default (EAD). However, the ASRF model only allows us to capture default dependencies between borrowers in a credit portfolio, even though a growing body of empirical studies confirms a positive relation between PD and LGD, ie, higher default rates seem to be accompanied by lower recovery rates (see, for example, Caselli et al 2008; Jacobs and Karagozoglu 2011). In light of these empirical findings, several models have been proposed that also allow for a stochastically dependent LGD (see, for example, Frye 2000; Hillebrand 2006; Pykhtin 2003).

A similar, yet less pronounced, strand of literature deals with the relation between PD and EAD. While empirical research has been conducted in the past to identify the factors driving borrowers’ EAD, and although there seems to be some anecdotal evidence that banks have carried out internal research to help the development of viable models to move forward, these findings have barely been considered for modeling

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1 Although structural models are almost exclusively analyzed and applied in a static context (as is done in the present paper, too), it should be mentioned that credit risk also has a dynamic dimension. Related literature on modeling the dynamics of credit risk using reduced-form models includes Koopman et al (2008) and Creal et al (2014).

2 See Basel Committee on Banking Supervision (2006, 2011). As the relevant rules do not differ between Basel II and III, we will speak simply of the “Basel framework”.
purposes in the academic literature (see, for example, Jacobs 2010; Jimenez et al 2009).

Nevertheless, shedding a brighter light on the effects of nondeterministic and dependent EAD in the context of credit risk modeling is crucial if a bank’s credit portfolio consists, at least in part, of revolving credit lines. For one thing, credit lines are often a major source of funding for corporate borrowers. For another, they work as liquidity insurance in economic downturns, particularly for US borrowers.3

We follow the empirical findings of Jimenez et al (2009) and propose a multifactor latent variable model with systematic and borrower-specific factors that can take into account the comprehensive dependence structure between PD, LGD and EAD. We extend the existing literature by combining and amending the approaches of Miu and Ozdemir (2006), for a stochastically dependent LGD, and Kupiec (2008), for a stochastically dependent EAD, in order to gain new insights regarding the joint impact of a stochastically dependent LGD and EAD on credit portfolio risk. Employing a simulation study, we set our results in contrast to those obtained using the ASRF model. Unlike some related contributions to this topic, we adjust our model so that the realized LGD and EAD values in it do not suffer from a selection bias. Several sensitivity and robustness analyses for different assumptions about the parameters are conducted to verify our results.

Since the supervisory authority is aware that ignoring LGD and EAD dependencies, as is done in the ASRF model, might lead to a significant underestimation of portfolio credit risk, banks applying the advanced internal ratings-based approach (A-IRBA) are required to estimate conservative values for LGD and EAD in order to reflect downturn scenarios and, thus, compensate for the lack of dependencies (see Basel Committee on Banking Supervision (2006, Sections 468 and 475) as well as EU Regulation No 575/2013, Articles 181 and 182). However, in addition to the regulatory requirements for the estimation of LGD and EAD within downturn scenarios, there is also a strong economic rationale for having a clear view of the effects of using downturn estimates for LGD and EAD in credit risk models. Since our analysis is at least partially driven by regulatory considerations, we propose methods to calculate values for downturn LGD and EAD based on the ASRF formula. As a result, we can capture the Basel framework’s underestimation of credit risk, due to its failure to take dependence into account when it only applies average values for LGD and EAD. We can derive suitable values for LGD and EAD in economic downturn scenarios if the ASRF model is applied.

Summarizing our main results, we find that a stochastically dependent EAD in conjunction with stochastically dependent PD and LGD significantly increases the

3 See, for instance, Sufi (2009) and Kashyap et al (2002). Implications for banks from a hereby potentially induced liquidity shock are surveyed in Basel Committee on Banking Supervision (2013).
credit risk of a portfolio. In most cases, separately estimating the risk markups for stochastically dependent LGD and EAD underestimates the inherent risk of a credit line portfolio. Further, we show that relative downturn markups on LGD and EAD greatly depend on asset correlations. Although credit portfolios with low asset correlations are typically considered to be less risky in terms of tail risk, we show that those portfolios might be particularly vulnerable to an underestimation of the additional capital charges for stochastically dependent LGD and EAD. This is due to the increasing relative importance of LGD and EAD risk as asset correlations diminish. Nevertheless, by adjusting LGD and EAD by their moments conditional on default, we demonstrate that our simulated results for the value-at-risk (VaR) and expected loss are considerably lower than in other related studies, indicating that the ASRF model does not underestimate the additional risk caused by a stochastically dependent LGD and EAD as much as one might expect. For portfolios exhibiting little systematic default dependence, the impact of a stochastically dependent EAD and LGD is higher. Depending on the chosen parameters, the estimated values for downturn LGD and EAD, which have to be used for the Basel framework’s capital formula, are significantly higher than their long-term averages. Further, we show that using separately calculated downturn estimates for LGD and EAD might not be a suitable solution in every case.

The remainder of this paper is organized as follows. Section 2 gives an overview of the related literature and presents the theoretical motivation for our modeling approach. The model is developed and specified in Section 3. In Section 4, based on a comprehensive simulation study, the impact of stochastically dependent LGD and EAD on credit risk is evaluated, and downturn estimates for LGD and EAD are derived. Section 5 concludes.

2 LITERATURE OVERVIEW AND REGULATORY BACKGROUND

In past years, several studies have empirically analyzed the potential factors driving LGD. Studies by Thorburn (2000), Hamilton et al (2004), Acharya et al (2007), Caselli et al (2008) and others provide ample evidence that LGD depends on both macroeconomic conditions and industry-specific parameters. In other words, when the economy worsens, LGD rates increase; therefore, bad years go along with, in addition to higher default rates, higher LGD values. This argument is reinforced by Frye (2000), Allen and Saunders (2003) and Hillebrand (2006), who state that the macroeconomic systematic factor that influences defaults also drives recoveries, since collateral is, like any other asset, sensitive to the state of the economy.

To capture the dependency between the PD and LGD in a factor-type latent variable model, Frye (2000, 2003), Tasche (2004), Giese (2005), Düllmann and Trapp (2005),
Rösch and Scheule (2005) and Jacobs (2011) use a single systematic factor that influences both the PD and LGD; this is in line with the aforementioned findings that the current macroeconomic conditions also drive the LGD.

In addition to these findings, Acharya et al (2004), Dermine and Neto de Carvalho (2006) and Khieu et al (2012) state that the LGD is influenced by borrower-specific characteristics such as firm size, the duration and intensity of the business relationship and default history. Consistently with these empirical results, Pykhtin (2003) argues that a borrower in financial distress reduces their efforts to maintain collateral, and that, therefore, the value of the collateral decreases, which implies a rising LGD in the case of a default event. Thus, a borrower-specific credit event can trigger both a rising PD and a rising LGD, irrespective of any market-wide systematic risks. As a result, a borrower-specific factor that influences the borrower’s PD as well as the LGD has to be taken into account, as is done by, for instance, Pykhtin (2003) and Miu and Ozdemir (2006), who include borrower-specific factors along with a common systematic factor in their models.

Compared with LGD drivers, debtors’ EAD is relatively unexplored, mainly due to the lack of appropriate data. Studies conducted by Agarwal et al (2006), Mester et al (2007), Jimenez et al (2009) and Norden and Weber (2010) state that borrowers running into financial distress and later defaulting on their credit lines draw down their lines more heavily than non-defaulting borrowers. The reason may be that firms running into default excessively draw down their credit lines in order to prevent insolvency by all means at their disposal. Therefore, these studies provide strong evidence that a debtor’s open exposure when default occurs and their default risk are closely related. Beyond these findings, Jimenez et al (2009) and Qi (2009) show that credit-line use is significantly negatively correlated with the business cycle. Hence, improving macroeconomic conditions leads to a decreasing use of credit lines. Tentative evidence for countercyclical EAD risk has been found by Jacobs (2010) and Araten and Jacobs (2001), indicating that banks clamp down on revolving lines of credit if they anticipate higher default rates and higher recovery risk when the economy worsens. This, in turn, could ex ante reduce EAD risk even if credit-line use may be higher. Overall, there seems to be mixed evidence regarding the correlation between PD and EAD, which may presumably depend on the type of portfolio. Switching the focus to the borrower’s characteristics, Sufi (2009), Jimenez et al (2009) and Jacobs (2010) discover that credit-line usage is influenced by borrower-specific factors such as profitability and the size of the firm.

4 Some of the latter studies use data on corporate credit lines for their empirical analyses, whereas others are based on data on consumer credit lines. However, the results are qualitatively the same for the different data sets in terms of the relation between PD and EAD.
As stated above, the credit risk model underlying the IRBA in the Basel framework does not consider stochastically dependent LGD and EAD, which is why the regulator requires banks applying the A-IRBA to derive downturn estimates for LGD and EAD. Calculating downturn estimates in a factor model for portfolio credit risk is a relatively new field of research. Regarding the calculation of a downturn LGD, Miu and Ozdemir (2006) assess by how much the LGD entering the Basel framework’s capital formula has to be increased compared with long-run mean LGD in order to account for the lack of correlation between the PD and LGD within the Basel framework’s IRBA. They find that, even under moderate parameter assumptions, a notable markup on the LGD is required, and therefore capital requirements are dramatically underestimated when LGD dependencies are ignored. Barco (2007) and, using a slightly different approach, Rösch and Scheule (2008) quantify the downturn LGD in different factor model settings and obtain qualitatively similar results regarding the LGD markups.

Empirical research done by Araten and Jacobs (2001), Taplin et al. (2007), Jacobs (2010), Witzany (2011) and others investigates the borrower’s expected open exposure at the time of default, using the loan equivalent quotient (LEQ) for revolving lines of credit as an essential part of the regulatory capital formula in practice. However, to the best of our knowledge, there exists no study examining the calibration and implications of downturn EAD or downturn LEQ by means of a factor-type latent variable model.

In order to take into account the aforementioned empirical findings, we have developed an integrated credit risk model for stochastically dependent PD, LGD and EAD. Our starting points are the models of Miu and Ozdemir (2006) and Kupiec (2008), which we combine and extend in several ways. As previously mentioned, Miu and Ozdemir (2006) model the dependencies between the PD and LGD in a factor model by incorporating borrower-specific factors as well as a common systematic factor. However, their proposed modeling framework is restricted to mirroring the dependencies between the PD and LGD, and it therefore neglects the correlation between PD and EAD carved out above. These dependencies between the PD and EAD are captured within a single factor model, developed by Kupiec (2008). To the best of our knowledge, this is the only existing theoretical approach that takes a nondeterministic

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5 Within the foundation IRBA, banks are generally required to use given values for LGD, provided by the regulator, instead of their own estimates. Moreover, Basel Committee on Banking Supervision (2006) explicitly requires banks to incorporate the present adverse dependency between the PD and LGD consistent with downturn conditions (see Basel Committee on Banking Supervision (2006, Sections 468 and 475) and EU Regulation No 575/2013, Articles 181 and 182).

6 Note that some of the aforementioned studies use different approaches for estimating the expected drawdown during the next period, namely the credit conversion or the EAD factor. Since these approaches are basically equivalent, we do not distinguish between them and only highlight that none of them deals with downturn conditions.
EAD and stochastically dependent PD and EAD in a factor model framework into account. Kupiec (2008) assumes that the PD, LGD and EAD are simultaneously driven by one common market-wide factor that reflects the current macroeconomic environment. However, that model does not consider borrower-specific dependencies; thus, the dependence structure is quite restricted due to the use of only one factor.

3 MODEL DEVELOPMENT AND DOWNTURN CALCULATIONS

3.1 Model design

In the framework of Black and Scholes (1973) and Merton (1974), it is assumed that the financial well-being of a firm is characterized by a latent asset value process. Analogously, a borrower \(i, i \in \{1, \ldots, n\}\), defaults if a latent variable \(\tilde{X}_{i,PD}\), which can be interpreted as borrower \(i\)’s standardized asset return, equals or falls below a certain threshold \(K_i\) at a predefined time horizon. Thus,

\[
\mathbb{I}_i := \begin{cases} 
1 & \text{if } \tilde{X}_{i,PD} \leq K_i, \\
0 & \text{otherwise.} 
\end{cases}
\]

\(K_i\) is chosen in such a way that the (unconditional) default probability of borrower \(i\) equals a predefined value \(PD_i\), i.e., \(K_i = \Phi^{-1}(PD_i)\), where \(\Phi^{-1}(\cdot)\) denotes the inverse of the standard normal cumulative distribution function. By definition, we model the default risk (as well as the LGD and EAD in the following) in a static context and abstain from the dynamics of credit risk, which is consistent with large parts of the literature dealing with structural portfolio models. Nevertheless, alternative model setups take into account these dynamics of default risk by explicitly including a time dimension in the models (see, for example, Das et al. 2007; Duffie et al. 2009; McNeil and Wendin 2007). Moreover, Creal et al. (2014) model a time-varying correlation structure between PD and LGD.

Following Miu and Ozdemir (2006), we define borrower \(i\)’s latent asset return variable \(\tilde{X}_{i,PD}\) as

\[
\tilde{X}_{i,PD} := q_{i,PD} \left( \beta_{PD} \tilde{Y} + \sqrt{1 - \beta_{PD}^2} \tilde{\epsilon}_{i,PD} \right) + \sqrt{1 - q_{i,PD}^2} \left( \theta_{i,PD} \tilde{I}_i + \sqrt{1 - \theta_{i,PD}^2} \tilde{\epsilon}_{i,PD} \right),
\]

where \(\tilde{Y}, \tilde{\epsilon}_{PD}, \tilde{I}_i\) and \(\tilde{\epsilon}_{i,PD}\) are assumed to be independent random variables following the standard normal distribution, and \(q_{i,PD}, \beta_{PD}, \theta_{i,PD} \in [0, 1]\). In (3.2), part A can be
interpreted as a systematic factor, while part B is a borrower-specific factor driving
the latent asset return variable. Part A is a composite systematic risk factor that can
be broken down into an overall systematic factor $\tilde{Y}$, which is also driving the LGD
and EAD on a systematic level (as will be seen in what follows), and a systematic
factor $\tilde{\nu}_{PD}$, which is only relevant for the latent asset return variable of all borrowers.
Analogously, part B consists of a factor $\tilde{I}_i$, which drives borrower $i$’s asset returns as
well as their LGD and EAD (again, this will be seen in what follows), and a factor
$\tilde{\varepsilon}_{i,PD}$, which is specific to borrower $i$’s asset returns. As a consequence, higher values
of $\beta_{PD}$ imply a more systematically driven PD, LGD and EAD dependency, which
is relevant for all borrowers. In contrast, higher values of $\theta_{i,PD}$ imply a stronger PD,
LGD and EAD dependency, which is specific for borrower $i$. Last, $\rho_{i,PD}$ determines
the systematic dependence between the asset returns of distinct borrowers and, thus,
their PDs.

To model a stochastically dependent LGD, we adopt the approach of Miu and
Ozdemir (2006), defining an additional latent variable

\[
\tilde{X}_{i,LGD} := \rho_{i,LGD}(\beta_{LGD}\tilde{Y} + \sqrt{1 - \beta_{LGD}^2}\tilde{\nu}_{LGD})
+ \sqrt{1 - \rho_{i,LGD}^2}(\theta_{i,LGD}\tilde{I}_i + \sqrt{1 - \theta_{i,LGD}^2}\tilde{\varepsilon}_{i,LGD}).
\]  

(3.3)

Analogously to (3.2), $\tilde{\nu}_{LGD}$ and $\tilde{\varepsilon}_{i,LGD}$ are independent and identically distributed (iid)
standard normal random variables, with the former being a systematic risk factor and
the latter being an idiosyncratic risk factor. The weighting parameters $\rho_{i,LGD}$, $\beta_{LGD}$ and
$\theta_{i,LGD}$ are again constrained to lie between 0 and 1. The correlation between the latent
variables for the PD and LGD for two distinct borrowers $i$ and $j$ is solely driven by
the systematic risk component, whereas the correlations between the latent variables
for the PD and LGD at the single-borrower level are also driven by a borrower-specific
risk component.

Following the related literature on credit risk, a sensible approach to modeling the
LGD distribution is the beta distribution using a quantile transformation.7 As stated
by, for instance, Pykhtin (2003) and Grundke (2005), when applying this technique
in combination with a latent variable factor model, one has to keep in mind that the
loss rates can, in principle, be calculated for the whole domain of borrowers in our
portfolio. This means that one would not differentiate between whether a borrower
has defaulted or not. More precisely, if we assume a nonzero correlation between
$\tilde{X}_{i,PD}$ and $\tilde{X}_{i,LGD}$, the simulated values for the mean and variance of the LGD for the
group of borrowers that defaulted do not coincide with their assumed first and second
moments ($\mu_{i,LGD}$ and $\sigma_{i,LGD}^2$). In fact, the first two moments of $\tilde{X}_{i,LGD} \mid \tilde{D}_i = 1$ are

7 In the sensitivity analysis, we also tried the inverse Gaussian, lognormal and gamma distributions.

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rather given by
\[ \mathbb{E}[\tilde{X}_{i,\text{LGD}} \mid \tilde{D}_i = 1] = -\text{Corr}(\tilde{X}_{i,\text{PD}}, \tilde{X}_{i,\text{LGD}}) \frac{\phi(K_i)}{\text{PD}_i} \] (3.4)
and
\[ \mathbb{V}[\tilde{X}_{i,\text{LGD}} \mid \tilde{D}_i = 1] = 1 - \text{Corr}(\tilde{X}_{i,\text{PD}}, \tilde{X}_{i,\text{LGD}})^2 \left( \frac{K_i}{\text{PD}_i} \frac{\phi(K_i)}{\text{PD}_i} + \left( \frac{\phi(K_i)}{\text{PD}_i} \right)^2 \right) \] (3.5)
where \( \phi(\cdot) \) denotes the standard normal density function.\(^8\) Not taking the conditional mean and variance of \( \tilde{X}_{i,\text{LGD}} \) into account may result in concentrated LGDs, as is shown later on. To avoid working with biased LGDs, we standardize the latent variable for LGD with its first two moments conditional on default, so that \( \tilde{\text{LGD}}_i \) is given by
\[ \tilde{\text{LGD}}_i := B^{-1} \left( 1 - \Phi \left( \frac{\tilde{X}_{i,\text{LGD}} - \mathbb{E}[\tilde{X}_{i,\text{LGD}} \mid \tilde{D}_i = 1]}{\sqrt{\mathbb{V}[\tilde{X}_{i,\text{LGD}} \mid \tilde{D}_i = 1]}} \right) ; a_{i,\text{LGD}}, b_{i,\text{LGD}} \right), \] (3.6)
with shape parameters \( a_{i,\text{LGD}} \) and \( b_{i,\text{LGD}} \), and the inverse Beta distribution \( B^{-1}(\cdot) \).\(^9\) We will address the consequences of this adjustment in our analysis in Section 4 and point out that the loss distribution as well as the tail risk will be significantly affected.

We model uncertainty about the end-period EAD by means of a stochastic loan equivalent factor \( \tilde{\text{LEQ}}_i \). The exposure in case of default for borrower \( i \) is a combination of both the known credit-line utilization at the beginning of the period (\( G_i \in [0, 1] \)) and an uncertain proportion of funds, which can additionally be drawn by the borrower up to the end of the period, i.e.,\(^10\)
\[ \tilde{\text{EAD}}_i := M_i (G_i + (1 - G_i) \tilde{\text{LEQ}}_i), \] (3.7)
where \( M_i \) denotes the granted maximum amount of funds.

In order to take the dependencies of \( \tilde{\text{LEQ}}_i \) with PD and LGD into account as well as model the EAD dependencies, we first set up a compound factor model with systematic and borrower-specific dependencies analogous to the PD and LGD:
\[ \tilde{X}_{i,\text{LEQ}} := q_{i,\text{LEQ}} (\beta_{\text{LEQ}} \tilde{Y} + \sqrt{1 - \beta^2_{\text{LEQ}}} \tilde{X}_{i,\text{LEQ}}) + \sqrt{1 - q^2_{i,\text{LEQ}}} (\theta_{i,\text{LEQ}} \tilde{I}_i + \sqrt{1 - \theta^2_{i,\text{LEQ}}} \tilde{\epsilon}_{i,\text{LEQ}}). \] (3.8)

\(^8\) For the derivation of the first two moments of a truncated normal distributed variable, we refer the reader to Cox and Wermuth (1992).

\(^9\) \[ a_{i,\text{LGD}} = \frac{\mu_{i,\text{LGD}} (1 - \mu_{i,\text{LGD}})}{\sigma_{i,\text{LGD}}^2} - \mu_{i,\text{LGD}} \quad \text{and} \quad b_{i,\text{LGD}} = \frac{a_{i,\text{LGD}}}{\mu_{i,\text{LGD}}} - a_{i,\text{LGD}}. \]

\(^10\) Note that, in the case of either a fully drawn credit line at the beginning of the period or a non-revolving type of loan, \( G_i \) equals 1, and, given our modeling approach, we are essentially dealing with a deterministic EAD that is no longer subject to uncertainty.
Similarly to the PD and LGD, \( \tilde{X}_{i,\text{LEQ}} \) and \( \epsilon_{i,\text{LEQ}} \) are iid standard normal random variables. The weighting parameters \( Q_{i,\text{LEQ}}, \beta_{1,\text{LEQ}} \) and \( \theta_{i,\text{LEQ}} \) are bounded between 0 and 1.

Following Jacobs (2010), we model \( \tilde{X}_{i,\text{LEQ}} \) by means of a beta distribution with borrower-specific expectation \( \mu_{i,\text{LEQ}} \) and variance \( \sigma_{i,\text{LEQ}}^2 \). Analogously to \( \tilde{L}_{\text{GDi}} \), we therefore employ a quantile transformation and calculate an adjusted draw rate, as given by\(^{11}\)

\[
\tilde{\text{LEQ}}_i := B^{-1}\left(1 - \Phi\left(\frac{\tilde{X}_{i,\text{LEQ}} - \mathbb{E}[\tilde{X}_{i,\text{LEQ}} | \tilde{D}_i = 1]}{\sqrt{\mathbb{V}[\tilde{X}_{i,\text{LEQ}} | \tilde{D}_i = 1]}}\right)\right): a_{i,\text{LEQ}}, b_{i,\text{LEQ}}. \tag{3.9}
\]

Given this definition, \( \tilde{\text{LEQ}}_i \) is bounded between 0 and 1. More precisely, we actually make two important assumptions with this definition. First, the LEQ’s having a maximum of 1 implicitly assumes that a borrower’s credit line does not include a tolerated overdraft by the bank, ie, the credit line is indeed capped at \( M_i \). Second, having a draw rate that cannot fall below zero ensures that \( \tilde{EAD}_i \) at the end-period is at least as large as the exposure of borrower \( i \) at the beginning of the period.\(^{12}\)

Up to now we have modeled PD, LGD and EAD with their systematic and borrower-specific dependencies. In the following, we will speak of the LGD (EAD) risk when considering a stochastic LGD (EAD) that is, to some extent, stochastically dependent at the systematic and/or borrower-specific level. We can now specify the realized losses for a single borrower using

\[
\tilde{L}_i := \tilde{L}_{\text{GDi}} \tilde{EAD}_i | \tilde{D}_i. \tag{3.10}
\]

We can also specify them for the whole credit portfolio with \( n \) borrowers over a time horizon of one year using

\[
\tilde{L}_{\text{PF}} := \sum_{i=1}^{n} \tilde{L}_i. \tag{3.11}
\]

### 3.2 The ASRF framework and the calculation of the downturn LGD and EAD

The values for the downturn LGD and EAD in our approach will be calculated in reference to the well-known ASRF model, which also constitutes a building block

\footnote{\(^{11}\)The expectation and variance of \( \tilde{X}_{i,\text{LEQ}} \), conditional on borrower \( i \) defaulting, are calculated along the same lines as (3.4) and (3.5).}

\footnote{\(^{12}\)Although this is a quite conservative assumption regarding EAD modeling, it is in line with the Basel framework’s requirements, as outlined in Section 474, which state that “for on-balance sheet items, banks must estimate EAD at no less than the current drawn amount, subject to recognizing the effects of on-balance sheet netting as specified in the foundation approach” (Basel Committee on Banking Supervision 2006, p. 104). However, these restrictions can be easily relaxed if, for instance, \( \text{LEQ}_i \) is not assumed to be beta distributed and, hence, is not bounded between 0 and 1.}
within the Basel framework’s IRBA to determine the capital requirements for credit risk. Under the assumptions of the ASRF model, the VaR of a credit portfolio given a confidence level \( \alpha \) with a fixed \( \text{LGD}_i \) of \( \mu_{i,\text{LGD}} \) and a fixed \( \text{EAD}_i \) of \( M_i (G_i + (1 - G_i) \mu_{i,\text{LEQ}}) \) can be calculated as

\[
\text{VaR}_\alpha^{\text{ASRF}} = \sum_{i=1}^{n} \Phi \left( \frac{\Phi^{-1}(\text{PD}_i) + \varphi_{i,\text{PD}} \Phi^{-1}(\alpha)}{\sqrt{1 - \varphi_{i,\text{PD}}^2}} \right) \text{LGD}_i \text{EAD}_i. \quad (3.12)
\]

As outlined above, banks employing the A-IRBA are obliged to use LGD and EAD estimates that appropriately reflect economic downturns, which we denote by \( \text{DLGD}_i \) and \( \text{DEAD}_i \), respectively.\(^{13}\) Taking these considerations into account, (3.12) can now be written as

\[
\text{VaR}_\alpha^{\text{IRBA}} = \sum_{i=1}^{n} \Phi \left( \frac{\Phi^{-1}(\text{PD}_i) + \varphi_{i,\text{PD}} \Phi^{-1}(\alpha)}{\sqrt{1 - \varphi_{i,\text{PD}}^2}} \right) \text{DLGD}_i \text{DEAD}_i. \quad (3.13)
\]

We want to contribute to the question of how these downturn values can be calculated. The basic idea is to assume that the true VaR is given by \( \text{VaR}_\alpha^{\text{Sim}} \) and then analyze which values of the downturn LGD and EAD have to be chosen in the ASRF framework to match this VaR, ie, substituting \( \text{VaR}_\alpha^{\text{IRBA}} \) with \( \text{VaR}_\alpha^{\text{Sim}} \) in (3.13) and solving for \( \text{DLGD}_i \) and \( \text{DEAD}_i \).

However, we face two problems. First, as our model’s simulated VaR is only given on a portfolio level, and an analytic solution of the portfolio loss distribution is not ascertainable, it is not feasible to assign correct \( \text{DLGD}_i \) and \( \text{DEAD}_i \) values in the case of a portfolio consisting of nonhomogeneous loans. This is because, in such a case, it is unclear how to adequately attribute the portfolio VaR to the heterogeneous individual loans with respect to borrower-specific downturn LGD and EAD values. Therefore, we need to assume a homogeneous portfolio of exposures for our calculation of the downturn LGD and EAD. Taking this into account, and henceforth dropping the subindex \( i \), (3.13) can be rewritten as

\[
\text{VaR}_\alpha^{\text{IRBA}} = \Phi \left( \frac{\Phi^{-1}(\text{PD}) + \varphi_{\text{PD}} \Phi^{-1}(\alpha)}{\sqrt{1 - \varphi_{\text{PD}}^2}} \right) n \text{DLGD} \text{DEAD}. \quad (3.14)
\]

Second, we cannot evaluate the individual effect of a downturn LGD and EAD simultaneously. To tackle this issue, we propose two different methods. In the first method, we calculate the downturn LGD and EAD separately. To derive DLGD, we simulate a portfolio VaR (denoted by \( \text{VaR}_\alpha^{\text{Sim,dLGD,\text{null}} \text{EAD}} \)), where we ignore the EAD risk, ie, where \( \text{LEQ} \) is fixed at its expected value conditional on default (\( \mu_{\text{LEQ}} \)). As a result, we ignore

\(^{13}\) There is specific guidance on the quantification of downturn LGD (see Basel Committee on Banking Supervision 2005), whereas this is not the case for downturn EAD.
the interplay between the LGD and EAD, which we take into account in the second mapping. The estimated downturn LGD for method 1 can be written as

$$DLGD^{m1} := \frac{\text{VaR}^{\text{Sim},\text{LGD},\text{dLEQ}}_\alpha}{\mathcal{P} n M (G + (1 - G) \mu_{\text{LEQ}})}, \quad (3.15)$$

where

$$\mathcal{P} = \Phi\left(\frac{\Phi^{-1}(PD) + \phi_{PD}^{-1}(\alpha)}{\sqrt{1 - \phi_{PD}^2}}\right).$$

The same approach can be applied to determine DEAD. In this case, a portfolio’s VaR (denoted by $\text{VaR}^{\text{Sim},\text{LGD},\text{dLEQ}}_\alpha$) is calculated without considering any LGD risk, ie, with a deterministic LGD fixed at its expected value $\mu_{\text{LGD}}$ conditional on default. The estimated downturn EAD for method 1 can then be written as

$$DEAD^{m1} := \frac{\text{VaR}^{\text{Sim},\text{LGD},\text{dLEQ}}_\alpha}{\mathcal{P} n \mu_{\text{LGD}}}. \quad (3.16)$$

In the second method, we only calculate a downturn LGD, which includes both the LGD and EAD risk as well as the interplay between the LGD and EAD.\textsuperscript{14} By calculating one downturn markup for both LGD and EAD, only the product of LGD and EAD risk is identified, and we cannot explicitly ascribe the downturn markup to its single components within our modeling approach. As a consequence, one can choose whether the markup is captured in a downturn LGD or a downturn EAD. However, if an interplay between the LGD and EAD is assumed, using the first mapping in order to calculate the downturn markups separately will underestimate the actual credit risk, as we shall see in Section 4.4. Hence, the choice of the mapping depends on the assumptions made regarding the relation between LGD and EAD. We use the VaR from the integrated model with a stochastically dependent LGD and EAD (denoted by $\text{VaR}^{\text{Sim},\text{LGD},\text{dLEQ}}_\alpha$) and determine the downturn LGD as

$$DLGD^{m2} := \frac{\text{VaR}^{\text{Sim},\text{LGD},\text{dLEQ}}_\alpha}{\mathcal{P} n M (G + (1 - G) \mu_{\text{LEQ}})}. \quad (3.17)$$

In addition to being generally required to have accurate values for LGD and EAD on the loan level, banks are supposed to be interested in having values for LGD and EAD on the portfolio level regarding downturn estimates for these risk parameters. Above, we assumed a homogeneous portfolio in terms of identical parameters for all borrowers. However, we could relax this assumption so that the borrowers have different values for PD and $\phi_{PD}$. In this case, we could assume a partially heterogeneous

\textsuperscript{14} This approach is in line with the regulatory requirements for retail portfolios as outlined in EU Regulation No 575/2013, Article 182(3) and Basel Committee on Banking Supervision (2006, Section 336).
### TABLE 1  Parameter settings for the basic setup.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td># of simulation trials $k$</td>
<td>10,000,000</td>
<td>$\theta_{\text{PD}} = \theta_{\text{LGD}} = \theta_{\text{LEQ}} := \theta$</td>
<td>0.40</td>
</tr>
<tr>
<td># of loans $n$</td>
<td>10,000</td>
<td>$\mu_{\text{LGD}}$</td>
<td>0.3900</td>
</tr>
<tr>
<td>$M$</td>
<td>1.00</td>
<td>$\sigma_{\text{LGD}}$</td>
<td>0.3027</td>
</tr>
<tr>
<td>$G$</td>
<td>0.50/0.00</td>
<td>$\text{LEQ}$ distribution beta</td>
<td></td>
</tr>
<tr>
<td>PD</td>
<td>0.01</td>
<td>$\beta_{\text{PD}} = \beta_{\text{LGD}} = \beta_{\text{LEQ}} := \beta$</td>
<td>0.73</td>
</tr>
<tr>
<td>$\theta_{\text{PD}}$</td>
<td>0.50</td>
<td>$\mu_{\text{LEQ}}$</td>
<td>0.4081</td>
</tr>
<tr>
<td>$\sigma_{\text{PD}}$</td>
<td>0.24</td>
<td>$\sigma_{\text{LEQ}}$</td>
<td>0.4058</td>
</tr>
</tbody>
</table>

Portfolio according to (3.13) and drop the subindex $i$ from either $\text{DLGD}_i$ or $\text{DEAD}_i$. This would allow them to be factored out of the sum and enable us to apply methods 1 and 2 analogously.

## 4 SIMULATION STUDY

### 4.1 Simulation setup

Before moving on to the simulation results, we first want to discuss the parameterization of our model; this is summarized in Table 1 for the basic parameter settings.

The initial model parameterization is based on the existing empirical literature. For most of the basic firm-value model parameters and the LGD parameters, we make use of Miu and Ozdemir (2006), who conducted an empirical analysis of real-world data to calibrate their model. For most of the parameters regarding the EAD, we rely on Jacobs (2010), who employed an extensive empirical analysis of the characteristics of revolving lines of credit.

In order to obtain reliable results for our high-dimensional model, we run 10,000,000 trials per simulation setup. The values for the parameters that determine the strength of the systematic dependence between the PD, LGD and EAD are all set to $\beta = 0.73$, as estimated by Miu and Ozdemir (2006). This corresponds to a moderate co-movement of the PD, LGD and EAD cycles, with a pairwise correlation of 0.5329. For the systematic dependence within the PD, we chose a value of $\theta_{\text{PD}} = 0.50$ (along the

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15 The parameter estimation for our model is beyond the scope of this paper. However, estimation procedures for conceptually related models can be found in Bierens (2007) and Bade et al (2011), who suggest a joint parameter estimation via the expected maximum likelihood algorithm. In addition, Jacobs (2011) proposes a two-factor structural model, which he calibrates to granular LGDs from Moody’s Ultimate Recovery Database via full information maximum likelihood.
same lines as Miu and Ozdemir (2006)), implying an asset return correlation of 0.25. Asset correlation is known to be an important driver of portfolio tail risk. However, the estimated values for asset return correlation vary considerably, due to the estimation technique as well as the parameterization of the model (see Düllmann et al 2010; Geidosch 2014). Further, since lower values for the asset correlation can be found in previous studies (see Frye and Jacobs 2012; Jacobs and Kiefer 2010), we test our model for different asset correlation values. The two parameters for the systematic dependence within the LGD and EAD, $\rho_{\text{LGD}}$ and $\rho_{\text{LEQ}}$, are both set to 0.24. Our model parameterization thus presumes a positive dependence between the PD and EAD. While there is very strong evidence of procyclicality in LGD, there is mixed evidence about EAD. For instance, Jacobs (2010) finds a slight countercyclicality in EAD and an inverse relation to PD. However, this does not invalidate our general model setup, since a negative correlation between PD and EAD could easily be captured in the factor model structure. Regarding the borrower-specific dependence between the PD, LGD and EAD, all parameters are set to $\theta = 0.40$. It should be noted that for $\beta_{\text{LEQ}}$, $\rho_{\text{LEQ}}$ and $\theta_{\text{LEQ}}$, empirical data is not available; we therefore approximate them by their LGD counterparts, estimated by Miu and Ozdemir (2006).

For LGD, the parameters $a_{\text{LGD}}$ and $b_{\text{LGD}}$ are chosen in accordance with Miu and Ozdemir (2006) in such a way that $\mathbb{E}[\text{LGD} \mid \tilde{D} = 1] = 0.39$ and $\mathbb{V}[\text{LGD} \mid \tilde{D} = 1] = 0.3072$. For LEQ, the parameters $a_{\text{LEQ}}$ and $b_{\text{LEQ}}$ are chosen in such a way that $\mathbb{E}[\text{LEQ} \mid \tilde{D} = 1] = 0.4081$ and $\mathbb{V}[\text{LEQ} \mid \tilde{D} = 1] = 0.4058^2$, as observed by Jacobs (2010).

4.2 Basic results

Based on the basic parameter settings presented in Section 4.1, we choose a stepwise approach, where we successively add the different risk elements in order to analyze their effect on the expected loss and VaR, and thereby on the portfolio credit risk. Our starting points are the analytically derived values for the ASRF model according to (3.12). In the first step, we simulate our model with deterministic values for LGD and EAD, ie, by fixing $\text{LGD}$ and $\text{LEQ}$ at their (conditional) expected values $\mu_{\text{LGD}}$ and $\mu_{\text{LEQ}}$, respectively. By simulating our model with a finite, but fairly large, number of loans, we slightly abandon the ASRF model’s infinite granularity assumption. This simulation is called Sim$\text{fLGD,LEQ}$. The second step involves separately introducing LGD and EAD risk. In the former case, denoted by Sim$\text{dLGD,LEQ}$, we simulate our model with a fixed $\text{LEQ}$ and a stochastically dependent $\text{LGD}$. In the latter, denoted by Sim$\text{fLGD,dLEQ}$, we keep $\text{LGD}$ fixed and treat $\text{LEQ}$ as a stochastically dependent variable. In the third step, we simulate our integrated model including LGD and

\[\text{Sim}_{\text{dLGD,LEQ}}\] can also be interpreted as the results for a portfolio with traditional loans instead of credit lines.
EAD risk, as presented in Section 3.1, with a stochastically dependent \( \bar{\text{LGD}} \) and \( \bar{\text{LEQ}} \); this is denoted by \( \text{Sim}_{d\text{LGD},d\text{LEQ}} \).\(^{17}\)

Besides this analysis of the impact of a dependent LGD and EAD, we are interested in two additional questions. First, we want to evaluate the impact of the adjustments with respect to \( \bar{\text{LGD}} \) and \( \bar{\text{LEQ}} \), as given by (3.6) and (3.9), respectively. We therefore simulate our integrated model with the unadjusted values for the portfolio’s LGD and EAD, which we denote by \( \text{Sim}_{\text{unadjusted}d\text{LGD},d\text{LEQ}} \). Second, we are interested in whether there is a difference between deterministic and stochastically independent values for LGD and/or EAD. To simulate the model with deterministic values for LGD and/or EAD, we therefore substitute the corresponding random variables (\( \bar{\text{LGD}} \) and \( \bar{\text{LEQ}} \)) with their expected values. Stochastically independent LGD and/or EAD can be modeled by setting their respective correlations parameters (\( \varphi \) and \( \theta \)) to zero.

In our basic simulation setup, we employ two different initial utilization rates. First, we assume that the initial utilization rate of the credit lines is 50%, i.e., \( G = 0.5 \). Given our model, this implies that each credit line’s finally realized EAD will take on values between 0.5 and 1.0 (with an expected value of \( M(G + (1 - G)\mu_{\text{LEQ}}) = 0.7041 \)). Thus, only the part above 0.5 of the granted credit line is subject to EAD risk. While this seems to be a sensible approach to analyzing the impact of LGD and EAD risk on a realistic credit (line) portfolio, it might not be suitable for comparing LGD and EAD risk. Therefore, we also employ an alternative simulation setup, where we set \( G = 0 \), i.e., where the full granted credit line is subject to EAD risk.

Table 2 shows the results for the above-mentioned simulation setups. We first investigate the setups where \( G = 0.5 \). In terms of expected loss, not much additional risk stems from the different elements. This negligible variation around zero in the expected losses for most model specifications can be attributed to simulation noise, since including additional dependencies between the major risk elements basically does not affect the expected losses. Only the integrated model shows a slight increase of 2.99% for the expected loss over the ASRF model, which a bank might consider when calculating the risk premiums charged for corresponding loans. This small increase in expected loss for our integrated model is mainly driven by the interplay between the LGD and EAD. Conceptually, considering dependencies between the PD and LGD as well as the PD and EAD in a factor model results in an interplay between the LGD and EAD, revealing an additional source of uncertainty.\(^{18}\) However, there is a

\(^{17}\) To clarify our notation, the prefixes “f” and “i” in the subindexes indicate whether the LGD and/or EAD are fixed at their (conditional) expected values (“f”) or treated as stochastically independent (“i”). The prefix “d” indicates whether the LGD and/or EAD risk are considered.

\(^{18}\) If the interplay between the LGD and EAD should be omitted for calculating regulatory capital requirements, however, a suitable methodology for downturn LGD estimations is presented in Section 4.4.
### TABLE 2  Simulation results for expected loss and VaR in the basic parameter setup.

#### (a) Expected loss

<table>
<thead>
<tr>
<th>Value</th>
<th>$G = 0.5$</th>
<th>$G = 0$</th>
<th>$G = 0.5$</th>
<th>$G = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASRF</td>
<td>27.46</td>
<td>15.92</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Sim$_{iLGD,iLEQ}$</td>
<td>27.43</td>
<td>15.91</td>
<td>-0.11</td>
<td>-0.03</td>
</tr>
<tr>
<td>Sim$_{iLGD,iLEQ}$</td>
<td>27.43</td>
<td>15.92</td>
<td>-0.11</td>
<td>0.00</td>
</tr>
<tr>
<td>Sim$_{dLGD,iLEQ}$</td>
<td>27.48</td>
<td>15.91</td>
<td>0.07</td>
<td>-0.03</td>
</tr>
<tr>
<td>Sim$_{dLGD,iLEQ}$</td>
<td>27.46</td>
<td>15.90</td>
<td>0.00</td>
<td>-0.08</td>
</tr>
<tr>
<td>Sim$_{iLGD,dLEQ}$</td>
<td>27.43</td>
<td>15.91</td>
<td>-0.11</td>
<td>-0.03</td>
</tr>
<tr>
<td>Sim$_{iLGD,dLEQ}$</td>
<td>27.47</td>
<td>15.91</td>
<td>0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>28.28</td>
<td>17.53</td>
<td>2.99</td>
<td>10.12</td>
</tr>
<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>44.86</td>
<td>34.83</td>
<td>63.36</td>
<td>118.84</td>
</tr>
</tbody>
</table>

#### (b) VaR$_{99.9\%}$

<table>
<thead>
<tr>
<th>Value</th>
<th>$G = 0.5$</th>
<th>$G = 0$</th>
<th>$G = 0.5$</th>
<th>$G = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASRF</td>
<td>503.87</td>
<td>292.06</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Sim$_{iLGD,iLEQ}$</td>
<td>503.30</td>
<td>292.85</td>
<td>-0.11</td>
<td>0.27</td>
</tr>
<tr>
<td>Sim$_{iLGD,iLEQ}$</td>
<td>500.70</td>
<td>292.61</td>
<td>-0.63</td>
<td>0.19</td>
</tr>
<tr>
<td>Sim$_{dLGD,iLEQ}$</td>
<td>553.76</td>
<td>319.63</td>
<td>9.90</td>
<td>9.44</td>
</tr>
<tr>
<td>Sim$_{dLGD,iLEQ}$</td>
<td>553.56</td>
<td>320.08</td>
<td>9.86</td>
<td>9.59</td>
</tr>
<tr>
<td>Sim$_{iLGD,dLEQ}$</td>
<td>516.45</td>
<td>326.99</td>
<td>2.50</td>
<td>11.96</td>
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<td>Sim$_{iLGD,dLEQ}$</td>
<td>520.59</td>
<td>328.18</td>
<td>3.32</td>
<td>12.37</td>
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<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>586.42</td>
<td>394.82</td>
<td>16.38</td>
<td>35.18</td>
</tr>
<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>898.07</td>
<td>726.48</td>
<td>78.24</td>
<td>148.74</td>
</tr>
</tbody>
</table>

A large increase in expected loss (63.36%) for the unadjusted model. This large deviation in expected loss between the adjusted and unadjusted models can be explained easily: taking a closer look at (3.4) and (3.5), it can be seen that the conditional expectation of $\tilde{X}_{i,LGD}$ typically takes values smaller than 0; likewise, the conditional variance of $\tilde{X}_{i,LGD}$ takes values smaller than 1 if a positive correlation between PD and LGD is assumed. This obviously contradicts the assumption that the variable obeys the

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19 The same considerations hold true for the EAD adjustment and should be applied analogously.
standard normal distribution, so the realized unadjusted LGD values may become considerably higher than expected, eg, than their long-term average. Taking these unadjusted LGD values as a basis for portfolio loss calculations according to (3.10) and (3.11), therefore, leads to losses that are generally distorted upward. Consequently, the expected loss, which we approximate by the average realized portfolio losses, is also distorted upward in the case where we assume an unadjusted LGD. Stated differently, the high increase for the expected loss in comparison to the ASRF model when employing an unadjusted LGD largely stems from the fact that too-high values of the LGD were assumed in the calculations; only a comparatively small portion is due to the LGD and EAD risk. These results are important for banks, which should incorporate expected loss into their risk-adjusted credit condition calculations as well as their loan loss provisions. This could, in turn, influence the valuation of a bank’s write-downs, which is required by the Capital Requirements Regulation (CRR) for IRBA institutions (see EU Regulation No 575/2013, Article 159; Basel Committee on Banking Supervision 2006, Season 384f). In addition, upward biased estimates for expected losses might also play a crucial role in impairing assets within new accounting approaches, such as International Financial Reporting Standard 9 (IFRS 9) and the current expected credit loss (CECL) model, which aim at replacing the existing incurred loss models with expected loss models.

For the VaR, unsurprisingly, we see no significant increase when looking at Sim_{LGDA,LEQ}. The concentration risk induced by relaxing the ASRF model’s infinite granularity assumption seems to be negligible from an economic point of view, because a relatively granular portfolio of 10 000 homogeneous loans is basically large enough to approach the limit of the ASRF formula. Replacing the fixed LGD and/or LEQ with their corresponding stochastically independent counterparts leads to only a slight increase in the VaR. In contrast, the introduction of a stochastically dependent LGD considerably increases the VaR by 9.90%. In contrast, the introduction of a stochastically dependent EAD induces a smaller, yet still considerable, increase of 2.50%. Therefore, at least for $G = 0.5$, the introduction of LGD risk has a larger impact than the introduction of EAD risk. For the integrated model, including both LGD and EAD risk, there is a remarkable increase of 16.38% for the VaR. Assuming that the dependencies between PD, LGD and EAD exist in our reference portfolio on a systematic as well as a borrower-specific level, and that, in addition, those dependencies are adequately captured by our model, the credit risk measured within the ASRF model is considerably underestimated in terms of VaR. It can be seen that summing the increments of a (solely) stochastically dependent LGD and EAD would only lead to an increase of 12.40% (9.90% + 2.50%). The remaining 3.98 percentage points of the integrated model’s increase obviously stem from the interplay between the LGD and EAD risk. Calculating the add-on risk charges for the LGD and EAD

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using separate models, and, thus, ignoring the interplay between the LGD and EAD, might therefore still underestimate the inherent credit risk.

In comparison with the expected loss, the VaR markup on the ASRF model is much more pronounced in the adjusted model. This is as expected, since the PD, LGD and EAD dependencies are generally considered to particularly affect the tail of the loss distribution and are far less likely to drive the expected loss. This observation is important, because even when the dependencies between those three risk components are negligible for the purpose of expected loss calculations, they are certainly not so for the allocation of capital to cover an unexpected loss.

Turning to the setups where $G = 0$, the results basically remain the same, with an overall higher amplitude in the relative changes to the ASRF model for simulations where a dependent EAD is involved. This is not surprising, since now the whole credit line is subject to EAD risk. For the integrated model, the VaR is 35.18% higher than in the ASRF model. However, the EAD risk now has a slightly higher impact than the LGD risk, which is due to the fact that $\sigma_{LEQ}$ is higher than $\sigma_{LGD}$. The order of a stochastically dependent LGD (respectively, EAD), concerning their influence on the VaR, switches at approximately $G = 0.1$. Thus, the markup for EAD risk is higher than the markup for LGD risk for setups where $G < 0.1$. If the parameterizations of the LGD and EAD risks are chosen to be similar to each other, we can conclude that their influence on the portfolio credit risk is comparable. However, assuming a parameterization in line with the empirical literature, the LGD risk basically has a greater influence than the EAD risk.

### 4.3 Sensitivity analysis

Sensitivity analyses are an important tool in testing the stability of some results as well as the impact of (small) changes in the inputs on the model’s outputs. Hence, they constitute an integral part of model risk assessment (see Board of Governors of the Federal Reserve System 2011). Therefore, we conducted a sensitivity analysis with $G = 0.50$. For each model parameter, we ran several simulations, in which the parameter under consideration was varied while the remaining ones were kept at their basic levels (see Table 1). In order to make the results comparable, we report them as changes of the VaR in relation to the ASRF model.\(^{21}\)

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\(^{20}\) In order to confirm this intuition, we conducted an additional simulation for a $G = 0$ setup, where we matched the LEQ distribution’s moments to those of the LGD distribution, i.e., $\mu_{LGD} = \mu_{LEQ} = 0.39$ and $\sigma_{LGD} = \sigma_{LEQ} = 0.3027$. The results show a comparable markup for the LGD and EAD risks of approximately ten percentage points.

\(^{21}\) The results for the expected loss as well as the economic capital and the expected shortfall are in line with the observations made in Section 4.2, i.e., while the expected loss does not vary (except for an expected slight increase in the integrated model), the economic capital and the expected shortfall behave just like the VaR. All results are available upon request.
Table 3 shows the results for the group of parameters for which a variation has only a minor impact on the relative difference between ASRF and Sim_{dLGD,dLEQ}. Those parameters are the number of loans in the portfolio, the PD and the distributions of LGD and LEQ, where we exchange the beta distribution for the lognormal, gamma and inverse-normal distributions. For all applied values of those parameters, the integrated model indicates a comparable markup of approximately 15–19% on the ASRF model’s VaR numbers. As already seen in the previous section, concentration risk is only of minor importance, leading to comparable results, irrespective of the number of

![Table 3](image-url)
It is interesting to see that a negative relation between PD and the markup, which is sometimes found in comparable models (that do not include any adjustment), is not manifest in the adjusted integrated model. The economic intuition behind this is that the within-PD dependencies are already captured in the ASRF model. Thus, a variation in the PD leads to rather small relative markups for our integrated model. For the distributions of the LGD and EAD, there is not much relevance concerning the implications for the VaR markups. Consequently, the shape of these distributions is only of minor importance, as long as their moments are calibrated correctly.

We now turn to the group of parameters that markedly influences the relative difference between the VaR for ASRF and Sim_{dLGD,dLEQ}. The results for this group are presented in Table 4. For $G$, the negative relation is reasonable, as lower values imply that a larger part (in relative terms) of the potential credit line is subject to EAD risk, as already described in Section 4.2. For $\beta$, it is obvious that a stronger co-movement of the PD, LGD and EAD cycles, indicated by higher $\beta$ values, leads to a higher markup over the ASRF model’s VaR. However, it is interesting to see that for low values of $\beta$, this markup can even become negative. Thus, when there is only a small dependency between the cycles of the PD, LGD and EAD (which, as outlined in Section 2, might not be a realistic assumption), the ASRF model is more conservative than our model. For $\rho_{LGD}$ and $\rho_{LEQ}$, higher values lead to a higher markup, as they induce a stronger relation between the single borrower’s $\hat{LGD}$ and $\hat{LEQ}$. Thereby, the amplitude is higher for $\rho_{LGD}$, since the LGD risk is more dominant for $G = 0.5$, as is shown in Section 4.2.

In contrast, the markup diminishes for higher values of $\rho_{PD}$; this is due to the fact that the additional risk stemming from higher $\rho_{PD}$ is also captured in the ASRF model. Therefore, the importance of the LGD and the EAD risk, which is captured in the markup, becomes less important in relative terms. This is a relevant finding especially for banks holding portfolios with low $\rho_{PD}$, as they might be tempted to neglect an accurate credit risk modeling due to their comparably low overall credit risk (in “the ASRF world”). However, the markup is relatively high for those banks in particular. This finding is also relevant since the regulatory authorities have recently paid a great deal of attention to the assessment of banks’ internal models for low-default portfolios.\footnote{For instance, the recent Basel Committee on Banking Supervision proposal aims at excluding some low-default portfolios from the scope of the application of the IRBA, and suggests applying parameter floors for PD and LGD in order to take into account the lack of data typically encountered for those portfolios (see Basel Committee on Banking Supervision 2016). Further, a benchmarking study of the European Banking Authority for low-default portfolios has been conducted (see European Banking Authority 2015).}
TABLE 4 Changes of VaR$_{99.9\%}$ relative to ASRF for various values of the parameters: group II.

(a) $G$

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>0.000</th>
<th>0.250</th>
<th>0.500</th>
<th>0.750</th>
<th>1.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>35.18</td>
<td>22.76</td>
<td>16.38</td>
<td>12.38</td>
<td>9.56</td>
</tr>
</tbody>
</table>

(b) $\beta$

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>0.000</th>
<th>0.250</th>
<th>0.500</th>
<th>0.750</th>
<th>1.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>−6.00</td>
<td>−3.31</td>
<td>4.05</td>
<td>17.26</td>
<td>35.37</td>
</tr>
</tbody>
</table>

(c) $\varphi_{PD}$

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>0.000</th>
<th>0.125</th>
<th>0.250</th>
<th>0.375</th>
<th>0.500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>100.21</td>
<td>50.23</td>
<td>34.26</td>
<td>24.35</td>
<td>16.38</td>
</tr>
</tbody>
</table>

(d) $\varphi_{LGD}$

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>0.000</th>
<th>0.125</th>
<th>0.250</th>
<th>0.375</th>
<th>0.500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>−4.71</td>
<td>5.14</td>
<td>17.15</td>
<td>30.08</td>
<td>44.28</td>
</tr>
</tbody>
</table>

(e) $\varphi_{LEQ}$

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>0.000</th>
<th>0.125</th>
<th>0.250</th>
<th>0.375</th>
<th>0.500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>8.33</td>
<td>12.28</td>
<td>16.50</td>
<td>21.15</td>
<td>25.87</td>
</tr>
</tbody>
</table>

(f) $\theta$

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>0.000</th>
<th>0.200</th>
<th>0.400</th>
<th>0.600</th>
<th>0.800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim$_{dLGD,dLEQ}$</td>
<td>29.10</td>
<td>24.48</td>
<td>16.38</td>
<td>0.77</td>
<td>−18.95</td>
</tr>
</tbody>
</table>

Values for the basic setup are underlined. All values given as percentages.
For \( \theta \), we can observe a negative relation between its value and the corresponding markup in the adjusted model. This result might be a bit striking, since rising correlations in the context of portfolio credit risk usually result in higher credit risks. To explain these counterintuitive results, we need to recall that the correlations between the PD, LGD and EAD for a single borrower are driven by a systematic component (\( \beta \)) and a borrower-specific component (\( \theta \)). Therefore, for higher values of \( \theta \), the relation between the PD, LGD and EAD on a borrower-specific level for a single borrower becomes predominant. However, since the LGD and EAD can only be observed if a default has actually occurred, the borrower-specific correlation between the PD and LGD as well as between the PD and EAD is already captured in their respective empirical estimates, ie, the values for \( \mu_{LGD}, \mu_{LEQ}, \sigma_{LGD} \) and \( \sigma_{LEQ} \). Actually, the adjustment corrects for this fact. This is a finding of great significance for banks as well as for regulatory purposes, because it implies that the ASRF model can lead to quite conservative capital estimates for credit risk if the underlying portfolio has rather high borrower-specific dependencies between the PD and LGD and/or the PD and EAD.

### 4.4 Downturn LGD and EAD

We now want to focus on the possibilities of taking into account a stochastically dependent LGD and EAD within the ASRF framework by means of downturn LGD and EAD. As pointed out before, LGDs and EAD are most often found to be positively correlated with default rates. Downturn LGD and EAD are supposed to consider these positive dependencies if constant values, rather than stochastically dependent PD, LGD and EAD, are assumed.

Unsurprisingly, it became obvious in the previous section that asset correlation as a key ingredient of most credit portfolio models is a substantial driver of credit portfolio tail risk.\(^{23}\) It can therefore be expected that asset correlation will heavily influence the range of the values of the downturn LGD and EAD. Although the calculation of the downturn LGD and EAD is crucial from an internal risk management perspective, this concept originally stems from regulatory considerations in the Basel framework for pillar I capital. For specific types of credit portfolios, the regulator provides binding information on how to derive the asset correlation, which, in turn, is used for the parameterization of the regulatory credit risk model. Against this background, we apply not only the asset correlation from our basic parameter setup (\( \varphi_{PD}^2 = 0.5^2 \)) but also the corresponding regulatory values, in order to derive estimates for the downturn LGD and EAD. More precisely, we also assume that our fictitious portfolio used in the previous sections represents a retail portfolio. For common retail

\(^{23}\) Asset correlation in our portfolio model is equal to \( \varphi_{PD}^2 \).
portfolios, the asset correlation is a function of the PD.\textsuperscript{24} If we still assume a PD of 1\%, this yields a regulatory asset correlation of $\rho_{\text{PD,retail}}^2 = 0.1216$. However, this value might not be appropriate for our specific portfolio, since for retail portfolios consisting of revolving lines of credit the regulatory framework prescribes a fixed asset correlation of $\rho_{\text{PD,creditline}}^2 = 0.04$ (see EU Regulation No 575/2013, Article 154(4); Basel Committee on Banking Supervision 2006, Section 328(2)). Table 5 contains the simulated VaR\textsuperscript{99.9\%} from the Sim\textsubscript{dlLGD,dLEQ} setup as well as the VaR\textsuperscript{IRBA} given by (3.14) (with the corresponding values of DLGD and DEAD, according to (3.15)–(3.17)) for both methods in the basic parameter setup presented in Section 4.1 and in the two regulatory setups. If we take a closer look at Table 5, two results are worth mentioning.

First, one can observe a significant difference between the values of DLGD and DEAD in the two methods in the basic scenario as well as in the regulatory setups. For instance, in the basic setup with $G = 0.5$, we obtain a downturn markup of 9.90\% on the expected LGD and 2.50\% on the expected EAD, which results in a 12.65\% higher VaR than in the ASRF model. However, this value is lower than the 16.38\% increase for the VaR in the Sim\textsubscript{dlLGD,dLEQ} scenario. The same is true if $G = 0$. In this case, the markup on the downturn LGD is 9.44\% and, hence, is of comparable size to that in the scenario with $G = 0.5$. The downturn EAD is now 11.96\% instead of 2.50\%, which is due to the fact that the loss distribution depends on the initial utilization rate of the credit line. Since our modeling approach assumes that the EAD at the end of the risk horizon is at least as large as the initial EAD, a lower initial credit-line utilization rate implies that a higher fraction of the EAD is uncertain from today’s perspective. This, in turn, makes the portfolio more vulnerable toward higher risk charges based on PD–EAD as well as PD–LGD dependencies.

In contrast, for method 2, in which only the values of DLGD are calculated, the markup is (by construction) the same as the relative difference between the values of VaR\textsuperscript{99.9\%} for Sim\textsubscript{dlLGD,dLEQ} and those for the ASRF model. How can the deviation between both approaches be explained? The reason is that method 1 captures both the PD–LGD and PD–EAD dependencies, but neglects the interplay between the LGD and EAD, whereas this interplay is additionally captured in method 2 and reflected in the values of the downturn LGD. The question, however, of which method is superior cannot be answered in general in light of these findings. In fact, the method chosen for (internal) portfolio credit risk management purposes should comply with the empirical findings and assumptions made on the relation between the LGD and

$$\rho_{\text{PD,retail}}^2 := 0.03 \frac{1 - \exp(-35\text{PD})}{1 - \exp(-35)} + 0.16 \left( \frac{1 - \exp(-35\text{PD})}{1 - \exp(-35)} \right);$$

see EU Regulation No 575/2013, Article 154(1) and Basel Committee on Banking Supervision (2006, Section 330).
TABLE 5  Simulation results for DLGD and DEAD in the basic parameter setup and for regulatory values of $\phi_{PD}$ for retail portfolios (of credit lines).

(a) $\phi_{PD}^2 = 0.5^2$ (basic setup)

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Change to ASRF (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G = 0.5$</td>
<td>$G = 0$</td>
</tr>
<tr>
<td>$\text{VaR}^{\text{SimLGD,dLEQ}}_{99.9%}$</td>
<td>586.42</td>
<td>394.82</td>
</tr>
<tr>
<td>$\text{DLGD}^{m1}$</td>
<td>0.4286</td>
<td>0.4268</td>
</tr>
<tr>
<td>$\text{DEAD}^{m1}$</td>
<td>0.7216</td>
<td>0.4569</td>
</tr>
<tr>
<td>$\text{VaR}^{\text{IRBA,m1}}_{99.9%}$</td>
<td>567.59</td>
<td>357.85</td>
</tr>
<tr>
<td>$\text{DLGD}^{m2}$</td>
<td>0.4539</td>
<td>0.5272</td>
</tr>
<tr>
<td>$\text{VaR}^{\text{IRBA,m2}}_{99.9%}$</td>
<td>586.42</td>
<td>394.82</td>
</tr>
</tbody>
</table>

(b) $\phi_{PD,retail}^2 = 0.1216$ (regulatory: retail)

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Change to ASRF (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G = 0.5$</td>
<td>$G = 0$</td>
</tr>
<tr>
<td>$\text{VaR}^{\text{SimLGD,dLEQ}}_{99.9%}$</td>
<td>315.07</td>
<td>222.29</td>
</tr>
<tr>
<td>$\text{DLGD}^{m1}$</td>
<td>0.4553</td>
<td>0.4548</td>
</tr>
<tr>
<td>$\text{DEAD}^{m1}$</td>
<td>0.7393</td>
<td>0.4915</td>
</tr>
<tr>
<td>$\text{VaR}^{\text{IRBA,m1}}_{99.9%}$</td>
<td>307.57</td>
<td>204.27</td>
</tr>
<tr>
<td>$\text{DLGD}^{m2}$</td>
<td>0.4898</td>
<td>0.5961</td>
</tr>
<tr>
<td>$\text{VaR}^{\text{IRBA,m2}}_{99.9%}$</td>
<td>315.07</td>
<td>222.29</td>
</tr>
</tbody>
</table>

(c) $\phi_{PD,creditline}^2 = 0.04$ (regulatory: credit lines)

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Change to ASRF (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G = 0.5$</td>
<td>$G = 0$</td>
</tr>
<tr>
<td>$\text{VaR}^{\text{SimLGD,dLEQ}}_{99.9%}$</td>
<td>155.94</td>
<td>114.48</td>
</tr>
<tr>
<td>$\text{DLGD}^{m1}$</td>
<td>0.4915</td>
<td>0.4915</td>
</tr>
<tr>
<td>$\text{DEAD}^{m1}$</td>
<td>0.7610</td>
<td>0.5373</td>
</tr>
<tr>
<td>$\text{VaR}^{\text{IRBA,m1}}_{99.9%}$</td>
<td>151.94</td>
<td>107.27</td>
</tr>
<tr>
<td>$\text{DLGD}^{m2}$</td>
<td>0.5453</td>
<td>0.6906</td>
</tr>
<tr>
<td>$\text{VaR}^{\text{IRBA,m2}}_{99.9%}$</td>
<td>155.94</td>
<td>114.48</td>
</tr>
</tbody>
</table>

In method 1 (m1), the interplay between the LGD and EAD is disregarded, implying that the DLGD and DEAD are derived separately. $\text{VaR}^{\text{IRBA,m1}}_{99.9\%}$ is, therefore, lower than $\text{VaR}^{\text{SimLGD,dLEQ}}_{99.9\%}$ in all cases. In method 2 (m2), the interplay between the LGD and EAD from the full model is explicitly taken into account and captured in the DLGD. $\text{VaR}^{\text{IRBA,m2}}_{99.9\%}$ as well as $\text{VaR}^{\text{SimLGD,dLEQ}}_{99.9\%}$ coincide by construction.
EAD. Nevertheless, neglecting the interplay between the LGD/EAD and method 1 even though it exists might lead to a significant underestimation of credit portfolio tail risk.

Second, one can pick up on a finding already made in Section 4.3: even though the values for the asset correlation in Table 5 are reported in descending order, the values of DLGD and DEAD are increasing, regardless of which method is used or which level of utilization of the initial credit line is assumed. On the one hand, lower asset correlations imply a lower VaR in absolute terms in both the ASRF model and the integrated model with stochastically dependent LGD and EAD. On the other hand, the relative importance of stochastically dependent LGD and EAD is much more pronounced if asset correlation in the portfolio is low. This results in larger deviations for the VaR between the ASRF model and our model. This finding is of great relevance from a regulatory perspective, since the reference value of $\rho_{PD}^2 = 0.5^2$ in the basic scenario is higher than the values prescribed by the regulator; however, the relative additional capital charges in terms of the downturn LGD and downturn EAD are much more pronounced if those values are used for the parameterization of the ASRF model. We therefore observe the highest DLGD and DEAD values in the scenario with the lowest asset correlation for portfolios of credit lines. In a nutshell, credit portfolios with lower asset correlations exhibit lower values, in absolute terms, of the VaR than credit portfolios with higher asset correlation, but the proper estimation of the LGD and EAD dependencies is much more crucial in the former case.

5 CONCLUSIONS

Recent literature has found strong evidence for the existence of both systematic and borrower-specific dependence between PD, LGD and EAD. Motivated by these findings, we propose a latent variable multifactor model for credit portfolio risk with systematic and borrower-specific factors. We hereby allow a comprehensive correlation structure between the PD, LGD and EAD to be taken into account. In order to gain insight into the model’s behavior, especially when compared with the ASRF model used by the regulatory authorities, we have conducted an extensive simulation study with several parameter variations.

Based on the chosen portfolio parameterization, we find that our integrated model for LGD and EAD risk markedly increases the VaR compared with the ASRF model. This increase is, however, less pronounced than comparable values from some related studies, since an adjustment to correct for a selection bias is incorporated into our model. A model taking into account the LGD and EAD risk separately might still significantly underestimate the true risk. For credit lines on which nothing has yet been drawn, the LGD and EAD risks are of comparable importance.
Further, we have conducted a sensitivity analysis. For the parameter $\rho_{PD}$, which drives the systematic within-PD dependence (and is already taken into account in the ASRF model), we find that, for lower values, our integrated model implies an increasing relative impact. In particular, banks with an indulgent loan portfolio, who have low dependencies compared with the ASRF model and, therefore, maybe do not care much about a sophisticated credit risk assessment, might be affected by the consequences of neglecting LGD and EAD risk.

Turning to the possibility of taking LGD and EAD risk into account in the ASRF model by using a downturn LGD and/or EAD, we have proposed two different mappings. The first involves a separate calculation of a downturn LGD and EAD. The second involves the calculation of merely a downturn LGD, which includes both the LGD risk and the EAD risk. We show that additional capital charges in terms of downturn LGD and EAD are decreasing in asset correlation. This finding is highly significant from a regulatory perspective, since the regulator assumes rather low values for the asset correlation. Against this background, accurate estimates of asset correlations are crucial, not only for measuring systematic default dependencies, but also for quantifying the additional impact of the dependencies between the PD, LGD and EAD on a portfolio’s credit risk.

**DECLARATION OF INTEREST**

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

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